EXTENDED ESSAY

PHYSICS

MECHANICS & FLUID DYNAMICS

Determination of Bicycle Drag Components Using the Coast-Down Model

How precisely can one determine the drag coefficient and separate the values of aerodynamic drag and rolling resistance for a bicycle at different speeds?

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1 Introduction

Cycling was one of my favourite hobbies as a child. Whether it was to school, in my neighbourhood, or on Wii Fit, I spent a lot of my time trying to optimise my bike rides. It took this personal hobby to gain an appreciation for bicycles as a physics invention, rather than a mere form of transportation. After studying mechanics and fluid dynamics, I wanted to investigate the behaviour of a bicycle travelling on a surface through a fluid and attempt to apply a model intended for larger vehicles to the case of bicycle.

Fluid dynamics is often viewed as one of the toughest subdisciplines of fluid mechanics, a prominent study within engineering. The coast-down test is a common procedure for a transportation vehicle to collect information about its condition. It is a process in which the vehicle is accelerated to an initial speed on a flat road, before neutrally coasting down to a low speed with no applied force. A model for the case of a bicycle would explain observations and allow cyclists to develop techniques which minimise drag components as much as feasible.

1.1 Aim

The aim of this study is to create a model using coast-down testing to calculate the distinct values of a bicycle's resistive force components, in order to understand how one can reduce them. This study will analyse the qualitative results to understand the behaviour of a body in motion through a fluid. Subsequently, it will touch upon how to make predictions at speed values beyond those covered in this investigation. This investigation is an attempt to calculate the value of the aerodynamic drag coefficient and rolling resistance. A secondary aim is to explain agreements and disagreements with the hypothesis, thus identifying strengths and weaknesses of the model.

1.2 Research Question

How precisely can one determine the drag coefficient and separate the values of aerodynamic drag and rolling resistance for a bicycle at different speeds?

2 Background Information

2.1 Air Resistance

Air resistance, or aerodynamic drag, is the friction opposing the motion of a body surrounded by a fluid. It is proportional to the square of the speed, thus dominating at high speeds. The relationship between air resistance and speed for bodies with high values of Reynolds number can be described as

$$F_a = kv^2 = \frac{1}{2}\rho A C_a v^2 \tag{1}$$

where ρ is fluid density, C_a is the drag coefficient, and v is speed.

2.1.1 Reynolds Number

In 1883, Osborne Reynold popularised the Reynolds number, which is a dimensionless quantity describing fluid behaviour and the relationship between the flow of a body and its velocity, length, density, and viscosity [3].

Cyclists have turbulent flow and high values of Reynolds number ($10^5 < Re < 10^6$) [5]. This allows us to make two assumptions:

1. Air resistance is proportional to the velocity squared. Eq. 1 holds.
2. The aerodynamic drag coefficient is constant and independent of speed.

2.2 Rolling Resistance

Rolling resistance is the component of vehicle drag which is often overlooked, particularly at high speeds. It is the opposing force which occurs when a body is rolling on a surface. Rolling resistance can be calculated by

$$F_r = C_r \cdot N = C_r \cdot mg \tag{2}$$

where C_r is the dimensionless rolling resistance coefficient, N is the normal reaction force, m is mass, and g is the gravitational field strength taken as 9.81 N kg^{-1}.

At low speeds, it is the rolling resistance which dominates. It can be approximated to be constant at speeds below 95 km h^{-1} [8], suitable for this speed range. At greater speeds, it is the coefficient of rolling resistance which varies.

2.2.1 Hysteresis

Rubber hysteresis is an interesting concept which opens for a more thorough understanding of why rolling resistance occurs, as it is one of the main causes. For every revolution of the bicycle wheel, the rubber releases less energy during unloading (area under curve B) than it uses to deform the rubber (area under curve A). Figure 2 shows a hysteresis loop, where the area between Curve A and B represents the energy lost. This is accounted for by an increase in internal energy and temperature.

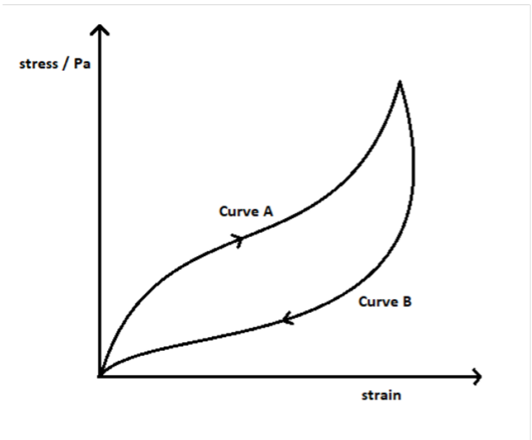

Figure 1: Created using Microsoft Paint

Stress (y-axis) is defined as the force per unit area (Pascals) and strain (x-axis) is defined as the change in length per unit length. The ratio between stress and strain is the Young's Modulus property, which describes the material's elastic properties. The advantage of using this quantity is that it allows for comparison between materials suitable for tyres. A smaller hysteresis loop is preferred for tyres as it implies the material is resilient, ensuring they do not disintegrate because of the increase in temperature.

In the following investigation, modern pneumatic tyres will be used as they minimises the rolling resistance. The results will also reflect everyday cases, rather than special cases with rigid and nonrubber wheels.

2.3 Other Forces

2.3.1 Gravity and Weight

For a vehicle descending down a slope, a component of its weight acts as a tractive force. The component of the weight parallel to the inclined plane can be calculated by

$$F_g = mg \cdot \sin\theta \tag{3}$$

where θ is the inclination angle.

The coast-down test on a descending slope should present results which match the test on a level ground, thus an alternative way to test the model. Recording the terminal speed and inclination angle is sufficient to make the desired calculations.

2.4 Conservation of Energy

Energy in a closed system is always conserved. This will act as a basis for the derivation of the model equation.

In this investigation, the forms of energy present in the moving bicycle are:

- Translational kinetic energy when in linear motion.
- Rotational kinetic energy is the kinetic energy due to the rotation of the wheels.

For testing done down a slope, the change in gravitational potential energy must also be accounted for.

2.5 Hypothesis

The rolling resistance will remain constant for all speeds examined in this investigation as the rolling resistance coefficient is not a function of speed. On the other hand, the aerodynamic drag will increase with speed.

As the Reynold's number of the body remains approximately the same throughout the coast-down test, speed does not affect the drag coefficient. Consequently, the coefficient of aerodynamic drag will assumed to be a constant value for the speeds covered in this investigation. This will yield values of air resistance which show its proportionality to the square of the speed.

The velocity vs time (vt) graphs for each trial should show a rectangular hyperbola as

$$m\frac{\mathrm{d}v}{\mathrm{d}t} = kv^2 - F_r \tag{4}$$

Eq. 4 shows that speed has an inverse proportional relationship to the root of the change in time. The beginning of the vt-graph should show a steep hyperbolic curve as the bicycle's deceleration also slows down as the speed decreases. The derivative, acceleration, should decrease with speed. The resultant force will approach the value of the constant rolling resistance. The graph is expected to tend towards a linear shape, as it approaches constant deceleration caused by rolling resistance and negligible air resistance.

3 Simulation and Model

3.1 Model Equation

Energy and power dissipation in the system will act as a starting point for the coast-down model equation. There is no change in potential energy on level ground. All kinetic energy will be transferred into work done against the aerodynamic drag and rolling resistance. Hence, the rate of change of kinetic energy is equal to the power consumed by the vehicle drag.

$$\frac{\mathrm{d}}{\mathrm{d}t}(KE) = P_a + P_r \tag{5}$$

where P_a is the power absorbed by air resistance and P_r is the power absorbed by rolling resistance. The time taken until the bicycle is brought to rest can be expressed as (Appendix A for detailed derivation)

$$t_0 = \left(\frac{2m^2}{\rho A C_a F_r}\right)^{\frac{1}{2}} \arctan\left[u\left(\frac{\rho A C_a}{2F_r}\right)^{\frac{1}{2}}\right] \tag{6}$$

where u is the initial speed of the bicycle.

The rationality of the expression may be evaluated by testing what happens at different limits. As the:

- area and rolling resistance increase, less time to be brought to rest.
- mass increases, more time to rest.

By introducing the dimensionless parameters, β, τ, and ν, it becomes possible to simplify the model equation. We define the coast-down parameter as [8]

$$\beta = u\left(\frac{\rho A C_a}{2F_r}\right)^{\frac{1}{2}} \tag{7}$$

The dimensionless time can be formulated as

$$\tau = \frac{t}{t_0} = 1 - \frac{\arctan\left[v\left(\frac{\rho A C_a}{2F_r}\right)^{\frac{1}{2}}\right]}{\arctan[\beta]} \tag{8}$$

Rearranging Eq. 8 to find speed as a function of dimensionless time

$$v = \frac{u}{\beta}\tan[(1-\tau)\arctan(\beta)] \tag{9}$$

Hence, the dimensionless speed ν is given by

$$\nu = \frac{v}{u} = \frac{1}{\beta}\tan[(1-\tau)\arctan(\beta)] \tag{10}$$

Now that dimensionless parameters have been introduced, further calculations and approximations are possible to make. They are also relevant in making predictions and applying this model to different cases (Section 5). The parameters allow comparisons between cases at different scales. The value of the coast-down parameter can be approximated as 1 for the range of speeds covered in this exploration [8]. It can be more accurately determined by taking the partial derivative of Eq. 10 w.r.t. β and $t_0(=\frac{t}{\tau})$ and solving them simultaneously. However, mathematics as such is beyond the scope of this investigation.

Rearranging Eq. 10 with the approximation that $\beta = 1$ (Section 3.5) gives another expression for the time until $v = 0$

$$t_0 = \frac{t\pi}{\pi - 4\arctan\left(\frac{v}{u}\right)} \tag{11}$$

Now, expressions for the drag coefficient and rolling resistance can be derived by dividing and multiplying equations.

Dividing Eq. 6 by Eq. 7 yields

$$\frac{t_0}{\beta} = \frac{\left(\frac{2m^2}{\rho AC_aF_r}\right)^{\frac{1}{2}}\arctan(\beta)}{u\left(\frac{\rho AC_a}{2F_r}\right)^{\frac{1}{2}}} \tag{12}$$

Therefore

$$C_a = \frac{2m\beta\arctan(\beta)}{t_0 u\rho A} \tag{13}$$

Multiplying Eq. 6 with Eq. 7 gives

$$t_0 \cdot \beta = \left(\frac{2m^2}{\rho AC_aF_r}\right)^{\frac{1}{2}}\arctan(\beta)\cdot u\left(\frac{\rho AC_a}{2F_r}\right)^{\frac{1}{2}} \tag{14}$$

Hence

$$F_r = \frac{mu \arctan(\beta)}{t_0 \beta} \tag{15}$$

The air density used was $1.209 kgm^{-3}$, calculated by

$$\rho = \frac{p}{R_{dryair} T} \tag{16}$$

It is important to note that integrating the functions in the derivation of Eq. 6 is only possible when assuming a high Reynolds number and a speed less than 95 kmh^{-1},ensuring that the rolling resistance coefficient may be treated as a constant.

3.2 Independent and Dependent Variables

Table 1: Independent and Dependent Variable

Independent variable Initial speed u / ms^{-1}	Before proceeding with data collection, the bicycle was accelerated to a fixed speed. Values of initial speed chosen for this investigation were approximately: 7, 6, 5, 4, 3 (m s^{-1}) with an uncertainty of ± 0.05 km h $^{-1}$ for direct measurements. The initial speed was predicted using a real-time speedometer. More accurate final values were found digitally using Tracker.
Dependent variable Time taken to decelerate t / s	The time taken for the bicycle to reach a final speed v was chosen as the dependent variable because it varies with different initial speeds and is a significant component in calculations. It was determined both manually and digitally, using a stopwatch and Tracker. Stopwatch uncertainty is ± 0.05s.

3.3 Constants and Conditions

Table 2: Constants and Conditions

Mass of the Bicycle and Rider / kg	Mass of the bicycle was determined using a luggage scale with an uncertainty of ±0.05 kg. Mass of rider was determined using a bathroom scale with an uncertainty of ±0.05 kg. These were kept constant as no load was added nor removed.
Distance / m	A fixed distance was chosen for consistent measurements of time taken to decelerate to final speed. This ensured that each trial was within video frame to be analysed using Tracker. The coast-down test was conducted on a horizontal pavement, which was measured to be 70.2 m with an uncertainty of ±0.05 m.
Tyre Pressure / Pa	Tyre pressure was controlled as it may impact the results of the coast-down test. This was done by controlling all affecting factors, mainly temperature. The bicycle did not travel for long enough distances for the tyre pressure to vary.
Temperature / °C	According to Gay-Lussac's law, pressure is directly proportional to temperature. As there were no environmental changes throughout the experiment, temperature remained constant. On the day of data collection, the temperature in Nordstrand, Oslo was 19°C ±0.05°C, determined by a wireless indoor/outdoor thermometer.
Atmospheric Pressure / Pa	Atmospheric pressure was used to calculate the air density. Air density was fixed because the air pressure did not change during data collection. The value of pressure was 101.30 kPa, obtained from [10]. This is approximately pressure under standard conditions, thus experimental results can be generalised.

Cross-Sectional Area / m^2	In all calculations, the cross-sectional area is taken as constant. It was kept constant by attempting to remain in the same position throughout the data collection. Additionally, this ensured that Assumption 3 (See Section 3.5) holds. As the calculation and measurement of surface area requires mathematics and digital tools beyond the scope of this investigation, a literature value was found from [2] according to a comparable position.
Clothing and Surface Area Material	In order to investigate the variation of aerodynamic drag with speed, its coefficient must remain constant. No clothing or surface area material was changed during data collection.

3.4 Tracker and Geogebra

Each trial was filmed and analysed using the video analysis tool, Tracker. The software was used to eliminate human error and obtain more accurate measurements of the physical quantities than those made directly. A calibration stick was used to calibrate the video scale, which is ratio of real distance to image distance in pixels between two points. The "Perspective" filter was used to ensure the angle from which the camera is placed did not affect the tracking and video analysis. Geogebra is a digital tool for mathematics, mainly used to plot graphs for each trial. Although this could also be done in Tracker, Geogebra has tools to create best-fit curves which are clearer to read and analyse.

3.5 Summary of Assumptions

1. High Reynolds number (turbulent flow) and $F_a \propto v^2$.
2. Maximum speed does not exceed 95 km h^{-1} (26 m s^{-1}).
3. Both coefficient of aerodynamic drag and rolling resistance are independent of speed.
4. Incline is $< 0.5\%$ and the test track can be approximated to be perfectly horizontal.
5. The coast-down parameter has a value of 1.

4 Experiment

4.1 Set Up and Procedure

Table 3: Equipment and Apparatus

Materials	**Quantity**	**Absolute Uncertainty**	**Percentage Uncertainty**
cPanasonic HC-V720	1	-	-
Tripod	1	-	-
Fixed distance	70.1 m	± 0.05 m	$\pm 0.07\%$
Stopwatch	2	± 0.05 ms	-
Speedometer	1	± 0.05 km h^{-1}	-
Nishiki Bushwacker 26"Mountain Bicycle	14.15 kg	± 0.05 kg	$\pm 0.35\%$
Rider	52.60 kg	± 0.05 kg	$\pm 0.10\%$

Experimental Procedure:

1. Record environmental data
   (a) Temperature, atmospheric pressure, wind velocity.
   (b) Note if the conditions are significant enough to impact the test results.
2. Carry out the coast-down test
   (a) Prior to the set distance within video frame, accelerate the bicycle to the fixed speed and ensure the speed is constant.
   (b) Once the bicycle enters the range, measure the initial speed, and do not pedal or apply any propulsive force.
   (c) Keep the bicycle in a straight path throughout the distance set for the test.
   (d) Once the bicycle reaches the end of the distance, measure the final speed and time taken to travel from start to end.
3. Repeat
   (a) Repeat Step 2 once again for each initial speed, this time in opposite direction. Execute 10 trials in total with 5 distinct values of initial speed.

Two classmates assisted by standing at the start and end points. Each started and ended the stopwatches simultaneously. Hence, two values of time were directly collected per trial, and their harmonic mean was calculated for one single time value.

4.2 Photograph of Equipment

Figure 2

Figure 3

Figure 4: Film recording

4.3 Experimental Results

Table 4: Raw Data from Direct Measurements

Coast-Down Test					
Trial number	Initial speed / *u* [m s^{-1} ±]	Final speed / *v* [m s^{-1} ±]	Time for Coast-Down / t [s]		Harmonic mean of time / t_{avg} [s]
1	8.639	7.333	8.830	9.930	9.348
	8.278	5.806	-	11.830	11.830
2	7.972	6.333	11.010	10.210	10.595
	7.833	5.444	11.160	12.030	11.579
3	7.139	5.056	13.280	12.200	12.717
	6.944	2.667	17.460	18.180	17.813
4	5.722	4.139	15.880	14.880	15.364
	5.444	2.306	20.730	21.600	21.156
5	4.500	2.250	22.920	22.070	22.487
	4.083	0.000	33.770	35.500	34.613

Table 5: Raw Data from Tracker

Coast-Down Test Tracker				
Trial number	Initial speed / u [ms^{-1}]	Final speed / *v* [ms^{-1}]	Time taken to decelerate/ *t* [s]	Harmonic mean of time / t_{avg} [s]
1	8.715	5.986	9.231	10.241
	7.533	4.298	11.500	
2	7.046	5.377	10.350	10.234
	7.809	5.010	10.120	
3	6.522	4.492	11.710	11.573
	4.274	3.345	11.440	
4	5.475	3.915	14.190	14.180
	5.143	2.751	14.170	
5	4.151	2.423	18.880	18.513
	3.883	1.691	18.160	

By comparing the data collected directly and digitally, the impact of human error may be examined. Raw data shows that time values measured using a stopwatch differ by 1 s depending on whether the observer is standing at the start or end point.

Each trial was conducted in both directions to account for wind speed. The wind speed measured in Nordstrand, Oslo was 3 m s^{-1} and wind gust of 5 m s^{-1}, which are within the constraints given by [7] for vehicle coast-down testing.

Towards the end of the experiment, the second direction experienced more headwind than the first direction, which is shown by a greater difference in final speed and time measurements. This effect is discussed is Section 4.6.

4.4 Calculations

The time taken for the bicycle to reach a state of rest was calculated using Eq. 6. The time value used was the harmonic mean of the digital values collected in each trial. The harmonic average was chosen as it disregards common denominators and ensures each data point weighs the same. The values of speeds used were their respective arithmetic means because the desired value is the most popular measure.

Table 6: Time for Coast-Down

Trial	Calculated Value of Time For Coast-Down $/t_0[s]$
1	36.376
2	45.883
3	57.737
4	49.545
5	46.582

Using Eqs. 1 and 13, the aerodynamic drag and its coefficient for each trial can be calculated as shown in Table 7 below.

Table 7: Experimental Aerodynamic Drag and Coefficients

Trial	Experimental Drag Coefficient $/C_a$	Experimental Drag $/F_a[N]$
1	0.694	11.712
2	0.602	8.493
3	0.0658	4.903
4	0.779	5.614
5	1.096	4.521

A cross-sectional area of 0.423 m^2 was used, which allows a coefficient value of 0.655 to be used as a reference literature value [2]. Table 7 shows accuracy at high speeds and greater error at lower speeds. This is discussed in Section 4.7.

Furthermore, the experimental value of rolling resistance was calculated using Eq. 15 as shown in the table below.

Table 8: Experimental Rolling Resistance

Trial	Experimental Rolling Resistance $/F_r[N]$
1	11.707
2	8.488
3	4.901
4	5.618
5	4.521

These calculations disagree with parts of the hypothesis, which is examined in the discussion and evaluation of this model.

4.5 Confirming accuracy

A simple way to evaluate the accuracy of the results is by comparing experimental and rough theoretical calculations.

Considering Trial 3 as an example.

$$F_{\text{net}} = -4.901 - 4.903 = -9.804 \text{ N}$$

By Newton's second law

$$a = \frac{F_{\text{net}}}{m} = \frac{-9.804}{66.750} = -0.147 \, \text{ms}^{-2}$$

Calculating the time taken to decelerate for Trial 3

$$t = \frac{v-u}{a} = \frac{3.919 - 5.398}{-0.147} = 10.068 \, \text{s}$$

The experimental time value collected for Trial 3 was 11.450 s, which is comparable to the theoretical value of 10.068 s. Close values indicate that the model successfully calculates certain unknown values and can distinguish between the two main resistive forces. The experimental value is slightly greater due to additional energy losses and errors discussed in Section 4.9.

4.6 Graphical Representation

For each trial, a graph of velocity versus time was plotted in both directions using Tracker and Geogebra (Appendix B). Tools as such were used to analyse the data and perform power regression to sketch an accurate graph including velocity values in between data points.

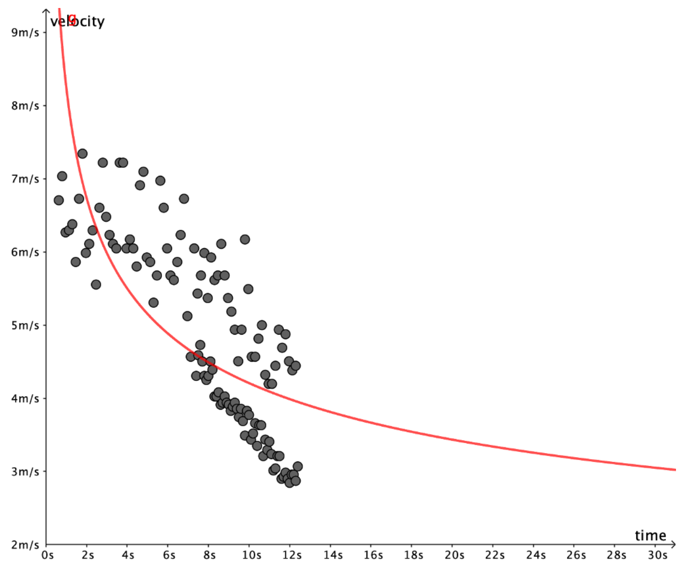

Figure 5: vt-graph for Trial 3 in Direction A.

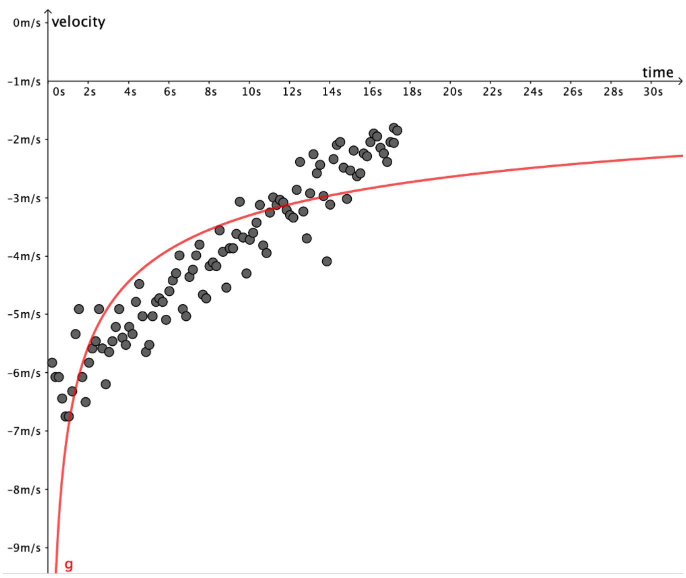

Figure 6: vt-graph for Trial 3 in Direction B.

The graphs show a hyperbolic curve (exponential decay) for the velocity, illustrating how the resistive forces experienced by the bicycle are not constant. We see that the derivative of the velocity-time graph decreases as the velocity approaches zero, which means the bicycle's deceleration is also decreasing with time. Therefore, by Newton's second law, the net force will decrease too.

Figure 5 and Figure 6 are approximately reflections of each other, which tells us that the environmental conditions and wind speed for this trial are close to negligible. The bicycle behaves approximately the same in both directions in Trial 3. Values used in the calculations for experimental drag and rolling resistance were points taken from this graphical analysis.

Observations can be made by examining at the variation in trials conducted in Direction A.

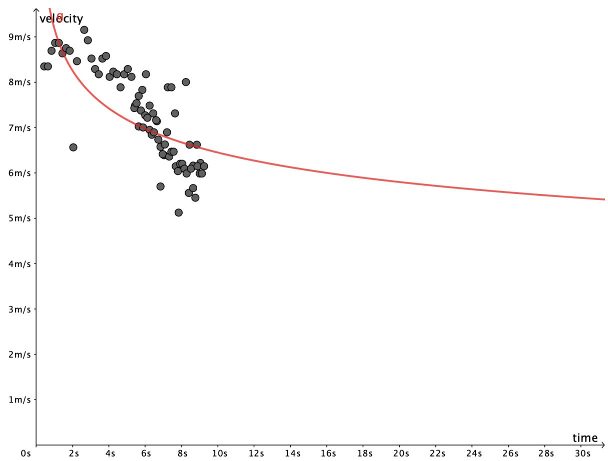

Figure 7: vt-graph for Trial 1A.

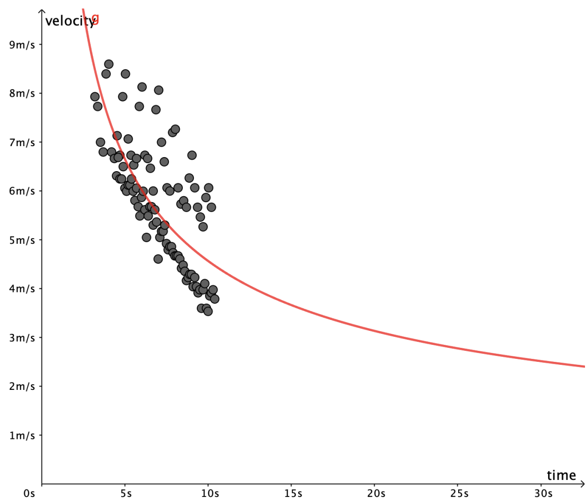

Figure 8: vt-graph for Trial 2A

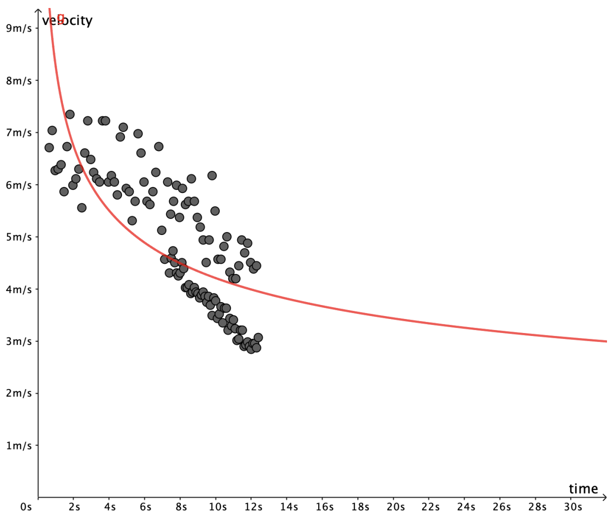

Figure 9: vt-graph for Trial 3A

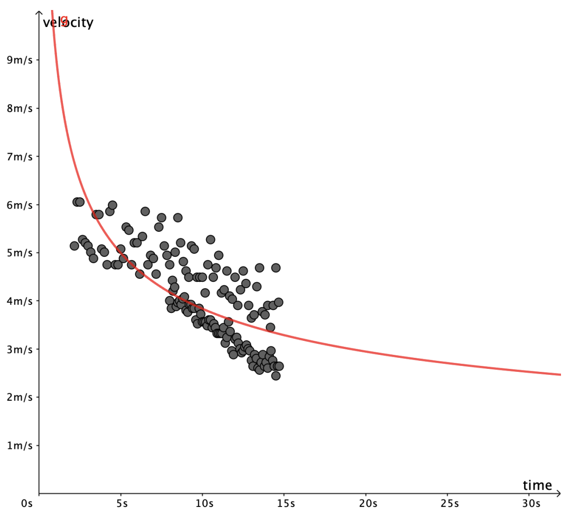

Figure 10: vt-graph for Trial 4A

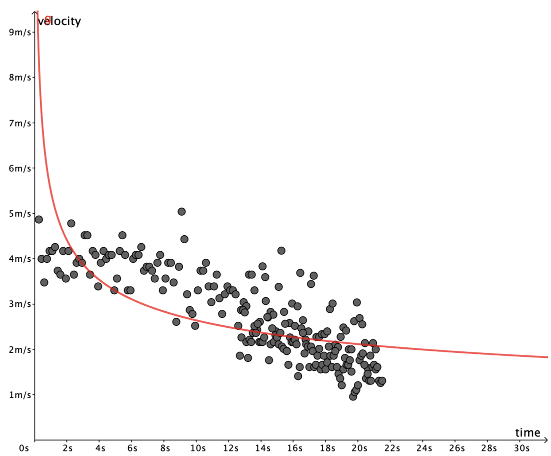

Figure 11: vt-graph for Trial 5A

As the initial velocity of the bicycle decreases:

- Initial gradient increases.
- Time taken for coast-down t_0 decreases.
- Time taken for the bicycle to reach an approximate constant speed decreases.
- Second derivative increases.

The second derivative of a vt-graph represents the physical quantity jerk, which is the rate at which the acceleration varies with time. This implies that as initial speed decreases,the net force reaches a constant value faster. Hence, it is shown that the constant rolling resistance dominates at low speeds, as aerodynamic drag rapidly becomes insignificant when the bicycle slows down.

4.7 Quantitative Observations

The data collected and calculations only agree with parts of the hypothesis.

For the first three trials, the experimental drag coefficient remains approximately constant. However, the last two slow trials experience an increase. This can be explained by the noted changes in environmental conditions. Data collection took a couple hours, and it got windier towards the last trials. As the wind speed, parallel to the motion of the bicycle, started to increase, the data collected became less precise and accurate. Thus, insufficient for this model which is sensitive to environmental factors at this scale.

It may be inferred that without these changes in the surroundings, the drag coefficient would follow the pattern shown by the first three trials and remain constant. Although the hypothesis was disproved, it is accountable and considered an exception to this case.

Calculations using raw data from Trials 4 and 5 can me made, which support the observation that random tailwind and headwind caused the inaccurate drag coefficients.

Table 9: Drag Coefficients for Trials 4 and 5

Drag Coefficient / C_a			
Trial 4		Trial 5	
Direction A	Direction B	Direction A	Direction B
0.553	1.054	0.856	1.387

The values in Table 9 show that the experimental coefficient in Direction A is lower than both the literature value and value calculated for Direction B. This quantitative observation explains why the experimental values break the trend of a constant drag coefficient. The results in Direction A imply the bicycle experiences a tailwind, as the wind cancels out parts of the air resistance. On the other hand, it experiences a headwind in Direction B. Hence, greater values of the drag coefficient are calculated due to stronger retarding forces.

This experimental flaw in the determination of the drag coefficient makes it difficult to confidently evaluate the hypothesis for the relationship between air drag and speed. The lack of applicable literature values of rolling resistance also excludes the possibility to calculate a fixed percentage error. Available research papers show a range from 2 N to 20 [8], which indicates numerical accuracy in order of magnitude.

Moreover, the experimental values of rolling resistance (Table 8) disagree with the hypothesis. The hypothesis was that rolling resistance remains constant, due to a speed-independent rolling resistance coefficient. Though the last three trials seem to project a constant force, the first two trials yield much greater values.

This is explained approximation made for the coast-down parameter. For higher speeds, such as those in trial 1 and 2, the optimal value of the coast-down parameter is not equal to 1. An operation including more complex mathematics would show more consistent and accurate results. Thus, I would conclude this model is not suitable for calculating values of rolling resistance at high speeds.

4.8 Uncertainties

The mass of the bicycle and rider were measured using a luggage scale and bathroom scale respectively. As the masses were added, their absolute uncertainties were added. Hence an uncertainty of ± 0.1 kg was assigned. Thus, the mass is expressed as

$$m = 66.750 \text{ kg} \pm \frac{0.100}{66.750} 100\%$$

Fluid density is given by

$$\rho = 1.209 \text{ kg m}^{-3} \pm \frac{0.050}{1.209} 100\%$$

The uncertainty for the final results depend on speed and time taken for coast-down. The aerodynamic drag coefficient and rolling resistance for Trial 3 is written as

$$C_a = 0.658 \pm \frac{0.085}{0.658} 100\%$$

and

$$F_r = 4.901N \pm \frac{0.043}{4.901} 100\%$$

Table 10: Percentage Uncertainty and Error for Drag Coefficients

Trial	Percentage Error [%]	Percentage Uncertainty [%]
1	5.95	11.712
2	8.09	8.493
3	0.46	4.903
4	15.57	15.37
	60.92	8.06
5	30.69	9.88
	111.76	6.09

Coefficients from Trials 4 and 5 are disregarded as the percentage error exceeds the percentage uncertainty. Therefore the first three trials are considered the only successful trials.

The percentage error was calculated by

$$\% \text{ error} = \frac{|\text{experimental value - literature value}|}{\text{literature value}} 100\%$$

4.9 Sources of Error

4.9.1 Systematic Errors

The main sources of systematic errors are equipment and procedure. They impede the experimental accuracy and cannot be reduced through repeated trials. However, acknowledging them help explain the results of the experiment.

1. **Non-Perfect Level Ground**

   The model used in this investigation assumes a perfectly horizontal ground. However, there was a small bump in the middle of the pavement, causing an additional loss in energy and slight change in rolling resistance. Rolling resistance is only constant if the surface remains the same.

2. **Camera Angle and Video Quality**

   The camera used recorded at 60 fps. A higher rate would increase experimental accuracy as more data points would be collected and plotted on the graph.

3. **Additional Frictional Losses**

   There are always additional losses of internal energy, especially as the apparatus was a bicycle. These were not accounted for and lead to an increase in experimental error.

4.9.2 Random Errors

Random errors decrease experimental precision. Several trials were carried through for each initial speed and the values were averaged for more accurate results.

1. **Changes in Wind Speed/Random Gusts**

   There were environmental changes which affected each individual reading. Towards the final trials, qualitative observations and data collected show significant wind velocity, which resulted in a greater difference in the net force acting on the bicycle in each direction.

2. **Positioning**

   The position and cross-sectional area of the rider varied throughout the experiment. Although it was attempted to remain in a similar position, this resulted in less precise data collection.

3. **Human Error**

   In measurements made directly with the stopwatches and speedometer, random errors occur from a too slow or fast reaction time. When collecting data through digital analysis, human error also occurs as I manually tracked the bicycle in each video frame.

5 Evaluation and Conclusion

5.1 Predictions and Applications

Ideally, this model could be used to calculate the aerodynamic drag and rolling resistance. However, from the results obtained in this investigation, it seems unfavourable to use the mathematical equation for rolling resistance. Although the model's graphical representation does show a constant rolling resistance, calculations did not yield consistent results.

However, an alternative application of the model can be made to predict the bicycle's behaviour at different speeds. At speeds within the limits which allow the drag coefficient and rolling resistance to be treated as constants, Newton's second law may be used to calculate the net force. Eq. 12 and Eq. 1 can be used to determine the aerodynamic component. Thus, one can successfully distinguish between air and rolling resistance by

$$F_{\text{net}} = F_a + F_r \tag{17}$$

On the other hand, it may also be necessary to first calculate the ideal value for the coast-down parameter. At higher speeds, the approximation of $\beta = 1$ will not hold, resulting in inaccurate results.

One can also apply this model different cases with additional retarding or tractive forces. Considering the case of an inclined path taken. By finding the vehicle's acceleration, Newton's second law can be used to calculate the net force acting on the body. The drag contributions may be determined by

$$F_{net} = F_a + F_r - F_g \tag{18}$$

The component of the weight is calculated using the angle of inclination. Therefore, only one of the resistive forces must be calculated to determine the other. An advantage of this application is that the model will be less sensitive to environmental factors, such as small wind speeds.

5.2 Strengths and Weaknesses

Although the strengths and weaknesses have been briefly mentioned in the discussions of observations and applications, they are important to recognise in order to make improvements.

5.2.1 Strengths

- **Simplified and accessible**

  The model has applied a complex procedure for cars and larger vehicles to the case of a bicycle. It is an educational experiment which does not require expensive or inaccessible equipment. Still, it provides thorough understanding of how a bicycle responds to motion through a fluid.

- **Everyday application of physics**

  The model reflects observations and expectations and clearly demonstrates the universality of physics. The results obtained from this investigation can be generalised and recognised in day-to-day cycling cases.

- **Experimental procedure**

  The experimental procedure is simple and feasible, as well as it was consistent and generally precise. A clear strength was conducting the experiment in both directions to account for external wind.

5.2.2 Weaknesses

- **Sensitive to small changes**

  The model is weak when it experiences any environmental wind resistance. It must be conducted under perfect conditions to yield desired results. This may be improved by conducting the experiment at higher speeds; however, it would require mathematical optimisation of the coast-down parameter. This may be improved by conducting the experiment down a slope, as the wind speed becomes negligible.

- **Mathematical Flaws**

  As frequently mentioned, the greatest shortcoming of the model lies in its simplification and the numerical approximation of the coast-down parameter. This results in particular weaknesses when calculating the rolling resistance. Improvements to this can be made by deriving another equation for the optimal value of the coast-down parameter.

5.3 Conclusion

Despite certain trials having a large uncertainty, the model derived in this investigation successfully managed to distinguish between the components of bicycle drag. Not all results agreed with the hypothesis but were explicable by considering the model's weaknesses. Final results showed an accurate drag coefficient value of 0.0658 ± 0.0003 with an error of ± 0.0032.

One can precisely distinguish between the values with negligible environmental factors and careful experimental procedure. Although, mathematical limitations challenge the potential of this model, it may be used to make general rough estimations of the bicycle drag components.

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Appendices

A Derivation of Equation 6

We use conservation of energy as an anchor and write it as the equation

$$\frac{\mathrm{d}}{\mathrm{d}t}(KE) = P_a + P_r \tag{I}$$

Using the chain rule and basic differentiation for the LHS w.r.t. time

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}(KE) &= \frac{\mathrm{d}}{\mathrm{d}t}(\frac{1}{2}mv^2)\\ &= \frac{\mathrm{d}}{\mathrm{d}v}(\frac{1}{2}mv^2)\cdot\frac{\mathrm{d}v}{\mathrm{d}t}\\ &= mv\cdot\frac{\mathrm{d}v}{\mathrm{d}t}\end{aligned} \tag{II}$$

Obtained from Eq. 1, while keeping Fr as a constant as it does not vary with speed

$$mv\cdot\frac{\mathrm{d}v}{\mathrm{d}t} = v(-\frac{1}{2}\rho AC_a v^2 - F_r) \tag{III}$$

Thus

$$-m\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{1}{2}\rho AC_a v^2 + F_r \tag{IV}$$

Solving the differential equation by rearranging and integrating both sides gives

$$\int_v^u -\frac{1}{\frac{\rho AC_a v^2}{2m} + \frac{F_r}{m}}\,\mathrm{d}v \;= \int_0^t 1\,dt \tag{V}$$

Obtaining

$$-\frac{\sqrt{2}m\arctan\left(\frac{\sqrt{\rho AC_a}v}{\sqrt{\rho AC_a F_r}}\right)}{\sqrt{\rho AC_a F_r}} = t + C \tag{VI}$$

Therefore

$$t + C = -\left(\frac{2m^2}{\rho AC_a F_r}\right)^{\frac{1}{2}}\arctan\left[v\left(\frac{\rho AC_a}{2F_r}\right)^{\frac{1}{2}}\right] \tag{VII}$$

To determine the integration constant, we solve Eq. VII for when $v = u$ and $t = 0$

$$C = -\left(\frac{2m^2}{\rho AC_a F_r}\right)^{\frac{1}{2}}\arctan\left[u\left(\frac{\rho AC_a}{2F_r}\right)^{\frac{1}{2}}\right] \tag{VIII}$$

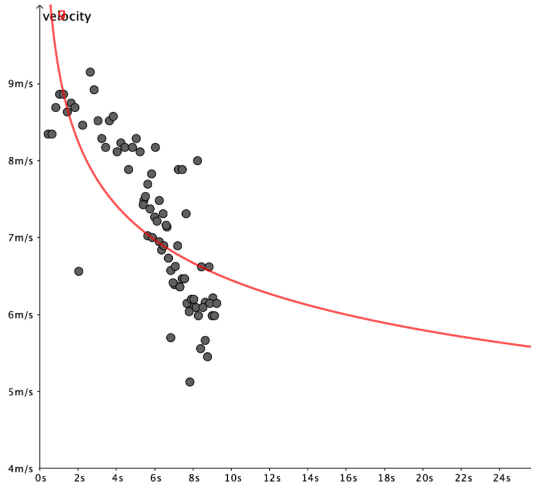
Substituting for C in Eq. VII gives

$$t = \left(\frac{2m^2}{\rho A C_a F_r} \right)^{\frac{1}{2}} \left[\arctan \left[u \left(\frac{\rho A C_a}{2F_r} \right)^{\frac{1}{2}} \right] - \arctan \left[v \left(\frac{\rho A C_a}{2F_r} \right)^{\frac{1}{2}} \right] \right] \tag{IX}$$

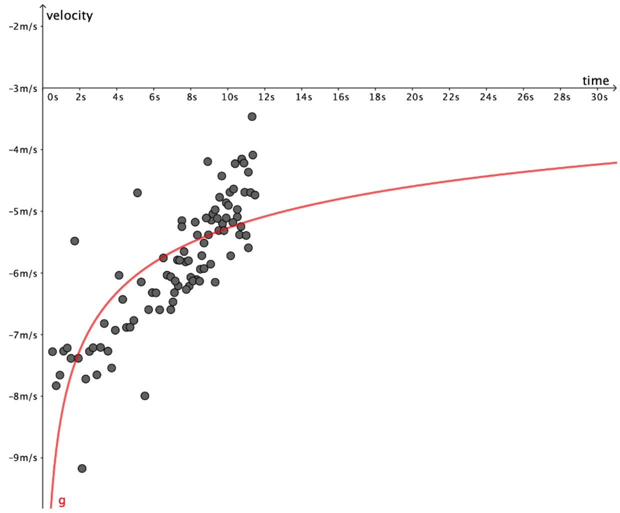
Thus, Eq. 6 is obtained by taking $v = 0$

$$t_0 = \left(\frac{2m^2}{\rho A C_a F_r} \right)^{\frac{1}{2}} \arctan \left[u \left(\frac{\rho A C_a}{2F_r} \right)^{\frac{1}{2}} \right] \tag{6}$$

B Graphs

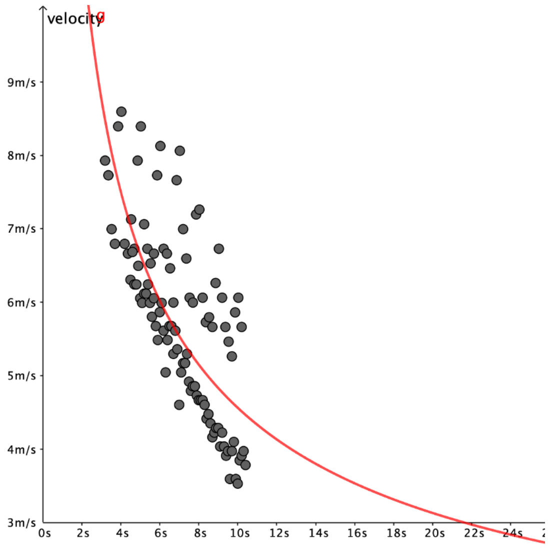


(a) Direction A

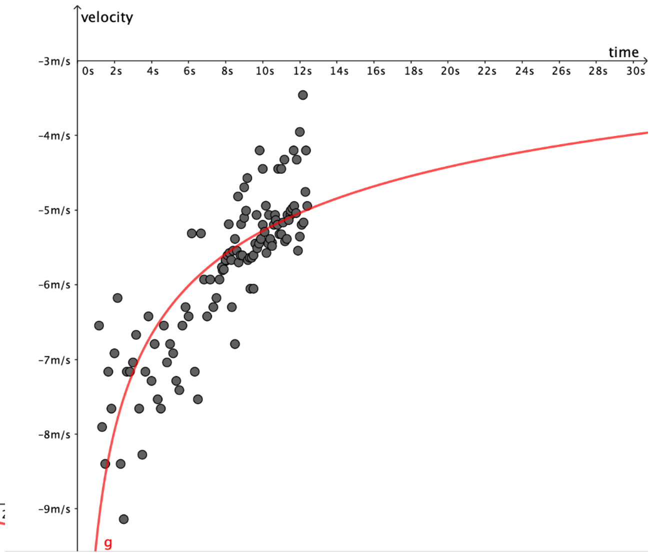


(b) Direction B

Figure 12: Trial 1

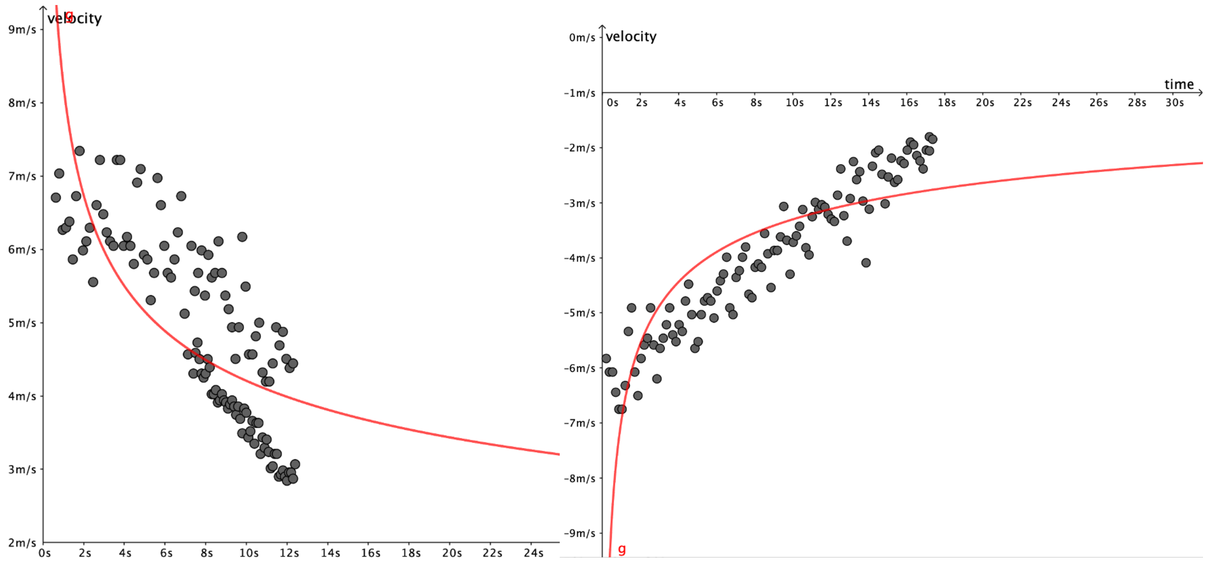


(a) Direction A



(b) Direction B

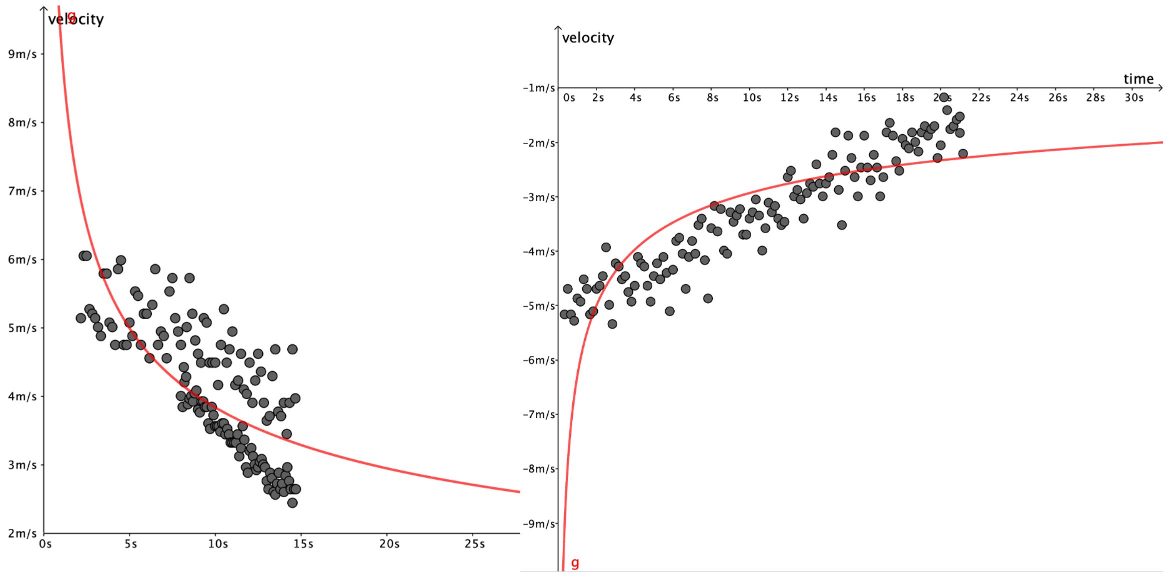
Figure 13: Trial 2



(a) Direction A

(b) Direction B

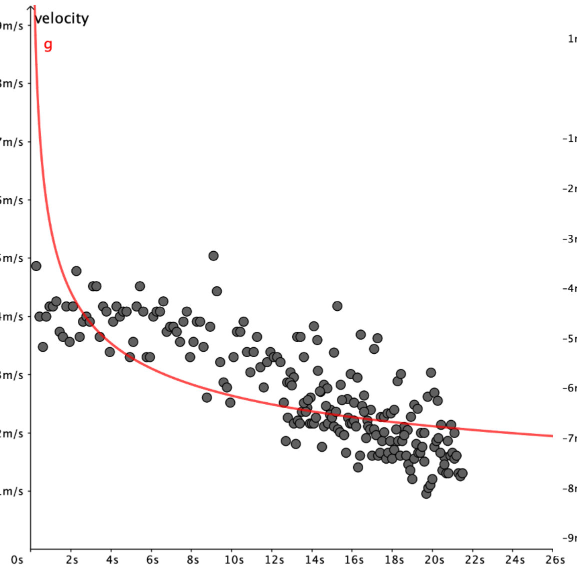
Figure 14: Trial 3



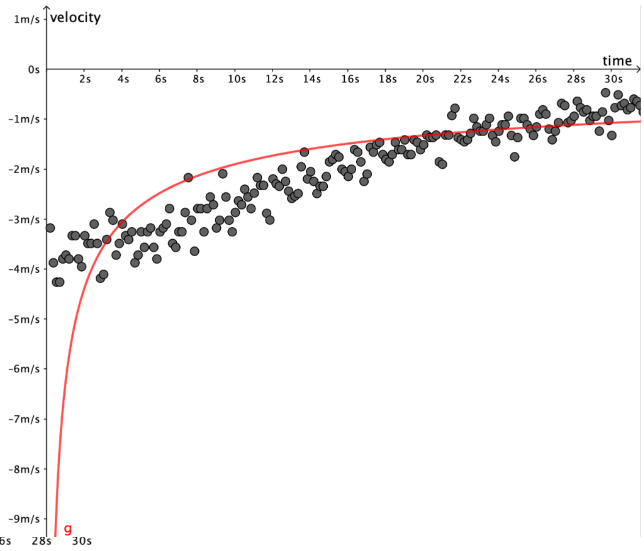
(a) Direction A

(b) Direction B

Figure 15: Trial 4



(a) Direction A



(b) Direction B

Figure 16: Trial 5

C Raw Data from Tracker

Trial 1a		Trial 1b		Trial 2a		Trial 2b		Trial 3a		Trial 3b		Trial 4a		Trial 4b		Trial 5a		Trial 5b	
t	v(t)	t	v(t)	t	v(t)	t	v(t)	t	v(t)	t	v(t)	t	v(t)	t	v(t)	t	v(t)	t	v(t)
0.440	8.348	0.520	-7.277	3.200	7.928	1.167	-6.546	0.633	6.708	0.200	-5.829	2.167	5.141	0.333	-5.162	0.267	4.866	0.200	-3.177
0.640	8.348	0.720	-7.829	3.367	7.728	1.333	-7.905	0.800	7.037	0.367	-6.075	2.333	6.052	0.500	-4.693	0.433	3.997	0.367	-3.874
0.840	8.693	0.920	-7.657	3.533	6.995	1.500	-8.399	0.967	6.268	0.533	-6.075	2.500	6.052	0.667	-5.162	0.600	3.476	0.533	-4.261
1.040	8.866	1.120	-7.268	3.700	6.795	1.667	-7.164	1.133	6.298	0.700	-6.443	2.667	5.272	0.833	-5.279	0.767	3.997	0.700	-4.261
1.240	8.866	1.320	-7.217	3.867	8.394	1.833	-7.658	1.300	6.380	0.867	-6.750	2.833	5.206	1.000	-4.869	0.933	4.171	0.867	-3.797
1.440	8.636	1.520	-7.385	4.033	8.594	2.000	-6.917	1.467	5.864	1.033	-6.750	3.000	5.141	1.167	-4.927	1.100	4.171	1.033	-3.719
1.640	8.751	1.720	-5.482	4.200	6.795	2.167	-6.176	1.633	6.728	1.200	-6.320	3.167	5.011	1.333	-4.517	1.267	4.258	1.200	-3.797
1.840	8.693	1.920	-7.383	4.367	6.662	2.333	-8.399	1.800	7.345	1.367	-5.338	3.333	4.881	1.500	-4.693	1.433	3.736	1.367	-3.332
2.040	6.563	2.120	-9.172	4.533	7.128	2.500	-9.140	1.967	5.987	1.533	-4.909	3.500	5.792	1.667	-5.162	1.600	3.649	1.533	-3.332
2.240	8.463	2.320	-7.720	4.700	6.728	2.667	-7.164	2.133	6.111	1.700	-6.075	3.667	5.792	1.833	-5.103	1.767	4.171	1.700	-3.797
2.640	9.154	2.520	-7.273	4.867	7.928	2.833	-7.164	2.300	6.296	1.867	-6.504	3.833	5.076	2.000	-4.693	1.933	3.563	1.867	-3.952
2.840	8.924	2.720	-7.212	5.033	8.394	3.000	-7.040	2.467	5.555	2.033	-5.829	4.000	5.011	2.167	-4.634	2.100	4.171	2.033	-3.332
3.040	8.521	2.920	-7.653	5.200	7.062	3.167	-6.670	2.633	6.605	2.200	-5.584	4.167	4.751	2.333	-4.458	2.267	4.779	2.200	-3.487
3.240	8.290	3.120	-7.208	5.367	6.728	3.333	-7.658	2.800	7.222	2.367	-5.461	4.333	5.857	2.500	-3.930	2.433	3.649	2.367	-3.487
3.440	8.175	3.320	-6.820	5.533	6.529	3.500	-8.276	2.967	6.481	2.533	-4.909	4.500	5.987	2.667	-4.986	2.600	3.910	2.533	-3.099
3.640	8.521	3.520	-7.268	5.700	6.662	3.667	-7.164	3.133	6.234	2.700	-5.584	4.667	4.751	2.833	-5.338	2.767	3.997	2.700	-3.487
3.840	8.578	3.720	-7.542	5.867	7.728	3.833	-6.423	3.300	6.111	2.867	-6.198	4.833	4.751	3.000	-4.223	2.933	3.910	2.867	-4.184
4.040	8.118	3.920	-6.929	6.033	8.127	4.000	-7.287	3.467	6.049	3.033	-5.645	5.000	5.076	3.167	-4.282	3.100	4.518	3.033	-4.106
4.240	8.233	4.120	-6.038	6.200	6.728	4.167	-6.793	3.633	7.222	3.200	-5.461	5.167	4.881	3.333	-4.517	3.267	4.518	3.200	-3.409
4.440	8.175	4.320	-6.429	6.367	6.662	4.333	-7.534	3.800	7.222	3.367	-5.216	5.333	5.532	3.500	-4.458	3.433	3.649	3.367	-2.867
4.640	7.887	4.520	-6.884	6.533	6.462	4.500	-7.658	3.967	6.049	3.533	-4.909	5.500	5.467	3.667	-4.751	3.600	4.171	3.533	-3.022
4.840	8.175	4.720	-6.880	6.700	5.996	4.667	-6.546	4.133	6.172	3.700	-5.400	5.667	4.751	3.833	-4.927	3.767	4.084	3.700	-3.719
5.040	8.290	4.920	-6.770	6.867	7.661	4.833	-7.040	4.300	6.049	3.867	-5.523	5.833	5.206	4.000	-4.634	3.933	3.389	3.867	-3.487
5.240	8.118	5.120	-4.700	7.033	8.061	5.000	-6.793	4.467	5.802	4.033	-5.216	6.000	5.206	4.167	-4.106	4.100	3.910	4.033	-3.099
5.440	7.484	5.320	-6.146	7.200	6.995	5.167	-6.917	4.633	6.913	4.200	-5.338	6.167	4.556	4.333	-4.223	4.267	4.171	4.200	-3.332
5.640	7.024	5.520	-7.994	7.367	6.595	5.333	-7.287	4.800	7.098	4.367	-4.786	6.333	5.337	4.500	-4.282	4.433	3.997	4.367	-3.409
5.840	7.830	5.720	-6.595	7.533	6.062	5.500	-7.411	4.967	5.926	4.533	-4.479	6.500	5.857	4.667	-4.634	4.600	4.084	4.533	-3.254
6.040	8.175	5.920	-6.319	7.700	5.996	5.667	-6.546	5.133	5.864	4.700	-5.032	6.667	4.751	4.833	-4.927	4.767	4.084	4.700	-3.874
6.240	7.484	6.120	-6.323	7.867	7.195	5.833	-6.299	5.300	5.308	4.867	-5.645	6.833	4.946	5.000	-4.458	4.933	3.302	4.867	-3.719
6.440	7.312	6.320	-6.597	8.033	7.261	6.000	-6.423	5.467	5.679	5.033	-5.523	7.000	4.881	5.167	-4.223	5.100	3.563	5.033	-3.254
6.640	7.139	6.520	-5.756	8.200	6.062	6.167	-5.311	5.633	6.975	5.200	-5.032	7.167	4.556	5.333	-4.517	5.267	4.171	5.200	-3.564
6.840	5.700	6.720	-6.036	8.367	5.729	6.333	-7.164	5.800	6.605	5.367	-4.786	7.333	5.532	5.500	-4.106	5.433	4.518	5.367	-3.254
7.040	6.390	6.920	-6.595	8.533	5.796	6.500	-7.534	5.967	6.049	5.533	-4.725	7.500	5.727	5.667	-4.399	5.600	4.084	5.533	-3.177
7.240	7.887	7.120	-6.319	8.700	5.663	6.667	-5.311	6.133	5.679	5.700	-4.786	7.667	5.141	5.833	-5.103	5.767	3.302	5.700	-3.564
7.440	7.887	7.320	-6.209	8.867	6.262	6.833	-5.929	6.300	5.617	5.867	-5.093	7.833	4.946	6.000	-4.341	5.933	3.302	5.867	-3.797
7.640	7.312	7.520	-5.151	9.033	6.728	7.000	-6.423	6.467	5.864	6.033	-4.602	8.000	4.751	6.167	-3.813	6.100	3.997	6.033	-3.254
7.840	5.124	7.720	-5.820	9.200	6.062	7.167	-5.929	6.633	6.234	6.200	-4.418	8.167	4.425	6.333	-3.754	6.267	4.084	6.200	-3.177
8.040	6.103	7.920	-6.213	9.367	5.663	7.333	-6.299	6.800	6.728	6.367	-4.295	8.333	5.011	6.500	-4.047	6.433	4.084	6.367	-3.099
8.240	8.002	8.120	-6.103	9.533	5.463	7.500	-6.176	6.967	5.123	6.533	-3.989	8.500	5.727	6.667	-4.693	6.600	4.258	6.533	-2.789
8.440	6.621	8.320	-6.107	9.700	5.263	7.667	-5.929	7.133	4.568	6.700	-4.909	8.667	5.206	6.833	-4.106	6.767	3.736	6.700	-3.487
8.640	6.160	8.520	-5.939	9.867	5.862	7.833	-5.805	7.300	6.049	6.867	-5.032	8.833	4.816	7.000	-3.813	6.933	3.823	6.867	-3.564
8.840	6.621	8.720	-5.928	10.033	6.062	8.000	-5.682	7.467	5.432	7.033	-4.357	9.000	4.621	7.167	-4.047	7.100	3.823	7.033	-3.254
9.040	6.218	8.920	-4.193	10.200	5.663	8.167	-5.188	7.633	5.679	7.200	-4.234	9.167	4.491	7.333	-3.520	7.267	3.736	7.200	-3.254
5.400	7.429	9.120	-5.143	4.500	6.306	8.333	-6.299	7.800	5.987	7.367	-3.989	9.333	5.141	7.500	-3.402	7.433	3.563	7.367	-2.867
5.520	7.536	9.320	-6.150	4.600	6.684	8.500	-6.793	7.967	5.370	7.533	-3.804	9.500	5.076	7.667	-4.165	7.600	3.910	7.533	-2.169
5.640	7.696	9.520	-5.311	4.700	6.243	8.667	-4.817	8.133	5.926	7.700	-4.663	9.667	4.491	7.833	-4.869	7.767	4.084	7.700	-3.022
5.760	7.376	9.720	-5.199	4.800	6.243	8.833	-5.188	8.300	5.617	7.867	-4.725	9.833	4.491	8.000	-3.578	7.933	3.302	7.867	-3.642
5.880	7.002	9.920	-4.862	4.900	6.495	9.000	-4.694	8.467	5.679	8.033	-4.173	10.000	4.491	8.167	-3.168	8.100	3.563	8.033	-2.789
6.000	7.269	10.120	-4.690	5.000	6.054	9.167	-4.570	8.633	6.111	8.200	-4.111	10.167	4.165	8.333	-3.637	8.267	3.910	8.200	-2.789
6.120	7.215	10.320	-4.638	5.100	5.991	9.333	-6.052	8.800	5.679	8.367	-4.173	10.333	4.751	8.500	-3.226	8.433	3.910	8.367	-3.254
6.240	6.948	10.520	-5.091	5.200	6.117	9.500	-6.052	8.967	5.370	8.533	-3.559	10.500	5.272	8.667	-3.989	8.600	3.476	8.533	-2.789
6.360	6.841	10.720	-5.248	5.300	6.117	9.667	-5.064	9.133	5.185	8.700	-3.927	10.667	4.491	8.833	-4.047	8.767	2.607	8.700	-2.557
6.480	6.895	10.920	-4.690	5.400	6.243	9.833	-4.200	9.300	4.938	8.867	-4.541	10.833	4.686	9.000	-3.285	8.933	3.823	8.867	-2.712
6.600	7.162	11.120	-4.364	5.500	5.991	10.000	-4.447	9.467	4.506	9.033	-3.866	11.000	4.946	9.167	-3.461	9.100	5.040	9.033	-3.177
6.720	6.734	11.320	-3.464	5.600	5.802	10.167	-4.941	9.633	4.938	9.200	-3.866	11.167	4.165	9.333	-3.344	9.267	4.432	9.200	-3.022
6.840	6.574	6.920	-6.062	5.700	6.054	10.333	-5.064	9.800	6.172	9.367	-3.620	11.333	4.230	9.500	-3.226	9.433	3.215	9.367	-2.092
6.960	6.414	7.040	-6.470	5.800	5.675	10.500	-5.435	9.967	5.493	9.533	-3.068	11.500	4.621	9.667	-3.696	9.600	2.867	9.533	-2.557
7.080	6.627	7.160	-6.130	5.900	5.486	10.667	-5.064	10.133	4.568	9.700	-3.682	11.667	4.100	9.833	-3.696	9.767	2.781	9.700	-3.022
7.200	6.895	7.280	-5.789	6.000	5.865	10.833	-4.447	10.300	4.568	9.867	-4.298	11.833	4.035	10.000	-3.402	9.933	2.520	9.867	-3.254
7.320	6.360	7.400	-5.794	6.100	5.991	11.000	-4.447	10.467	4.815	10.033	-3.720	12.000	4.491	10.167	-3.285	10.100	3.302	10.033	-2.867
7.440	6.467	7.520	-5.249	6.200	5.612	11.167	-4.323	10.633	5.000	10.200	-3.601	12.167	3.905	10.333	-3.050	10.267	3.736	10.200	-2.634
7.560	6.467	7.640	-5.653	6.300	5.045	11.333	-5.064	10.800	4.321	10.367	-3.428	12.333	4.230	10.500	-3.344	10.433	3.736	10.367	-2.712
7.680	6.146	7.760	-6.268	6.400	5.486	11.500	-5.064	10.967	4.197	10.533	-3.124	12.500	4.621	10.667	-3.989	10.600	3.910	10.533	-2.402
7.800	6.039	7.880	-5.801	6.500	5.675	11.667	-4.200	11.133	4.197	10.700	-3.818	12.667	4.360	10.833	-3.578	10.767	3.389	10.700	-2.557
7.920	6.200	8.000	-6.071	6.600	5.675	11.833	-4.323	11.300	4.444	10.867	-3.948	12.833	3.905	11.000	-3.109	10.933	3.041	10.867	-2.789
8.040	6.200	8.120	-6.130	6.700	5.297	12.000	-3.952	11.467	4.938	11.033	-3.254	13.000	3.644	11.167	-3.285	11.100	3.389	11.033	-2.479
8.160	6.093	8.240	-5.176	6.800	5.612	12.167	-3.458	11.633	4.691	11.200	-2.994	13.167	3.710	11.333	-3.168	11.267	3.649	11.200	-2.169
8.280	5.986	8.360	-5.385	6.900	5.360	12.333	-4.200	11.800	4.876	11.367	-3.124	13.333	4.295	11.500	-3.402	11.433	3.128	11.367	-2.324
8.400	5.558	8.480	-6.134	7.000	4.603	7.800	-5.763	11.967	4.506	11.533	-3.037	13.500	4.686	11.667	-3.520	11.600	2.781	11.533	-2.324
8.520	6.093	8.600	-5.719	7.100	5.045	7.900	-5.794	12.133	4.382	11.700	-3.081	13.667	3.775	11.833	-3.461	11.767	3.215	11.700	-2.884
8.640	5.665	8.720	-5.515	7.200	5.171	8.000	-5.668	12.300	4.444	11.867	-3.211	13.833	3.710	12.000	-2.640	11.933	3.389	11.867	-3.019
8.760	5.452	8.840	-5.108	7.300	5.171	8.100	-5.605	7.400	4.306	12.033	-3.298	14.000	3.905	12.167	-2.522	12.100	3.302	12.033	-2.196
8.880	6.146	8.960	-5.381	7.400	5.297	8.200	-5.574	7.500	4.588	12.200	-3.341	14.167	3.449	12.333	-2.992	12.267	3.302	12.200	-2.294
9.000	5.986	9.080	-5.857	7.500	4.919	8.300	-5.668	7.600	4.729	12.367	-2.864	14.333	3.905	12.500	-2.874	12.433	3.215	12.367	-2.342
9.120	5.986	9.200	-5.040	7.600	4.793	8.400	-5.543	7.700	4.503	12.533	-2.386	14.500	4.686	12.667	-3.050	12.600	2.520	12.533	-2.001
9.240	6.146	9.320	-4.974	7.700	4.856	8.500	-5.385	7.800	4.306	12.700	-3.235	14.667	3.970	12.833	-3.402	12.767	2.867	12.700	-2.245
		9.440	-5.115	7.800	4.856	8.600	-5.543	7.900	4.250	12.867	-3.697	8.000	4.004	13.000	-2.933	12.933	3.041	12.867	-2.440
		9.560	-4.772	7.900	4.730	8.700	-5.700	8.000	4.306	13.033	-2.924	8.100	3.844	13.167	-2.757	13.100	2.954	13.033	-2.586
		9.680	-4.427	8.000	4.667	8.800	-5.605	8.100	4.503	13.200	-2.254	8.200	4.204	13.333	-2.816	13.267	3.649	13.200	-2.538
		9.800	-5.313	8.100	4.667	8.900	-5.605	8.200	4.391	13.367	-2.581	8.300	4.285	13.500	-2.405	13.433	3.649	13.367	-2.489
		9.920	-5.108	8.200	4.667	9.000	-5.102	8.300	4.025	13.533	-2.435	8.400	3.884	13.667	-2.757	13.600	3.302	13.533	-1.952
		10.040	-4.904	8.300	4.603	9.100	-5.007	8.400	4.025	13.700	-2.971	8.500	3.964	13.833	-2.992	13.767	2.433	13.700	-1.659
		10.160	-5.721	8.400	4.414	9.200	-5.668	8.500	4.081	13.867	-4.091	8.600	4.004	14.000	-2.757	13.933	2.607	13.867	-2.196
		10.280	-5.181	8.500	4.477	9.300	-5.637	8.600	3.912	14.033	-3.117	8.700	3.924	14.167	-2.640	14.100	3.832	14.033	-2.050
		10.400	-4.227	8.600	4.351	9.400	-5.637	8.700	3.940	14.200	-2.337	8.800	4.044	14.333	-2.229	14.267	3.594	14.200	-2.245
		10.520	-4.970	8.700	4.162	9.500	-5.605	8.800	4.025	14.367	-2.094	8.900	4.084	14.500	-1.818	14.433	2.704	14.367	-2.489
		10.640	-5.381	8.800	4.225	9.600	-5.448	8.900	3.940	14.533	-2.045	9.000	3.804	14.667	-2.874	14.600	2.826	14.533	-2.342
		10.760	-4.152	8.900	4.288	9.700	-5.511	9.000	3.912	14.700	-2.484	9.100	3.764	14.833	-3.520	14.767	2.765	14.700	-2.342
		10.880	-4.218	9.000	4.288	9.800	-5.448	9.100	3.828	14.867	-3.019	9.200	3.924	15.000	-2.522	14.933	2.396	14.867	-2.147
		11.000	-5.392	9.100	4.036	9.900	-5.385	9.200	3.884	15.033	-2.532	9.300	3.924	15.167	-1.877	15.100	3.379	15.033	-1.854
		11.120	-5.592	9.200	4.225	10.000	-5.196	9.300	3.940	15.200	-2.191	9.400	3.844	15.333	-2.288	15.267	4.178	15.200	-1.806
		11.240	-4.695	9.300	4.036	10.100	-5.291	9.400	3.856	15.367	-2.630	9.500	3.844	15.500	-2.640	15.433	2.826	15.367	-1.708
		11.360	-4.087	9.400	3.910	10.200	-5.574	9.500	3.743	15.533	-2.581	9.600	3.604	15.667	-2.992	15.600	1.966	15.533	-1.757
		11.480	-4.736	9.500	3.973	10.300	-5.448	9.600	3.856	15.700	-2.240	9.700	3.524	15.833	-2.464	15.767	2.581	15.700	-2.001