Physics

Extended Essay

Elastics, Energy, and Efficiency

Words: 3933

Acknowledgements

Firstly, I would like to show my gratitude to Chris Hamper, my physics teacher, for supporting me on the journey through this investigation. I would also like to thank my father for explaining the chemistry of rubber bands so that I was able to deeper understand the topic.

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I. Introduction

Energy can be stored and released by stretching a rubber band and letting it go. Its elastic construction has made it an excellent use as a motor for toy vehicles such as wind-up cars and balsa wood airplanes. By using methods like twisting or winding you can use a rubber band to store energy and release it to make the toys move. A marvellous use of an everyday object to power small vehicles. Nevertheless, could a rubber band create enough energy for a bigger vehicle? And what method would be the most efficient?

There are two main methods to store energy through deforming a rubber band. One way is through twisting it. This method is often used for balsa wood toy airplanes, where the rubber band is attached to two fixed points, one in the back and one on the propeller which will be turned to twist the rubber band. The work done to twist it will then be stored in the rubber due to conservation of energy. Another method of deforming a rubber band is by winding it around an axis. In this situation the rubber band is attached to one fixed point at one end, and a rod on the other. The rod is the axis which will be turned to wrap the rubber band around. This method is often used for toy cars and is in that situation attached to the back rod connected to the two wheels. In this essay I will explore the two different methods of deforming a rubber band to store and release energy.

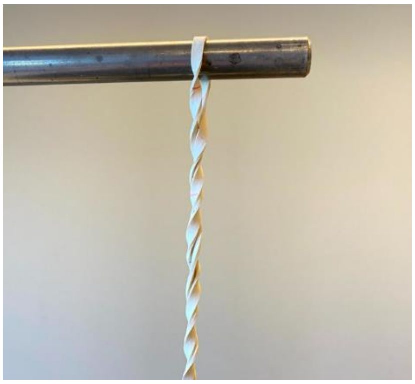

Figure 2: Twisted rubber band

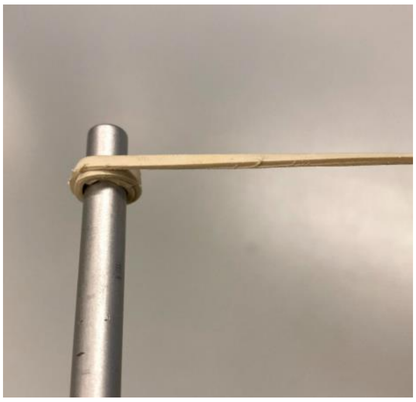

Figure 1: Winded rubber band

II. Research question:

What is the most efficient method for storing and releasing energy in a twisted rubber band compared to a wound rubber band?

The approach to investigate this question will be to measure the energy released by the rubber band after having been deformed. Then the work done to deform the rubber band will be measured to compare the two methods and their efficiency.

III. Theoretical approach

When work is done to deform a rubber band, the kinetic energy is transferred to potential energy within the band. As it goes back to its original shape the energy gets released in the form of kinetic energy again. The more energy that is used to deform the band the more force is required to further deform it. Rubber bands consist of long chains of molecules called polymers. When the rubber band is in its original shape, the polymers are all tangled up. This makes it easy to stretch. Whereas when the rubber band is stretched, what happens is that the polymers, that are flexible, are also starting to stretch and straighten out (see figure 1). When they are completely stretched out it takes more force to keep stretching the rubber band as you will have to stretch out along the already untangled polymers. As the rubber band is being stretched and becomes thinner, the molecule chains

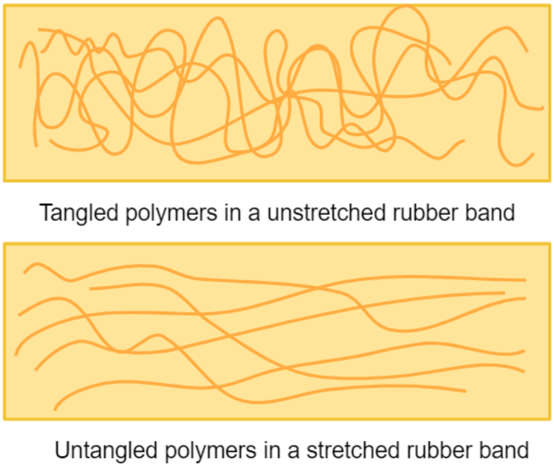

Figure 3: Polymers in a stretched and unstretched rubber band

move closer together and some of the work done to stretch the band is being transferred to heat energy within the rubber (Gibbs, 2013).

Due to rubber bands consisting of a material like this, the work done to stretch it is not the same as released when unstretched. Instead, the extension and release of a rubber band can have different paths due to parts of the energy used transferring to heat energy. This phenomenon is called elastic hysteresis and occurs when the energy used to stretch a rubber band is greater than the energy released when going back to original form. Looking at figure 4 we can see the graph of force against extension when stretched and released. The area between the stretched and released graph is the energy lost to heat produced. Whereas the space underneath the released graph is the energy left to be let out when going back to the original position.

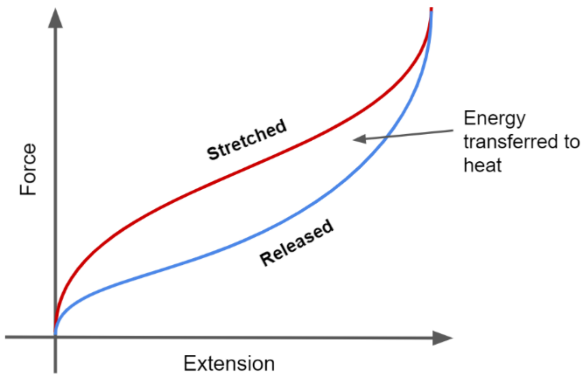

Figure 4: Force vs. Extension graph of a rubber band being loaded and unloaded

This is the theory behind simply extending a rubber band by its two ends. However, if the rubber band is deformed using a method, such as twisting or winding, would the theory still be the same? Energy will always be stored and released when stretching a rubber band whereas some parts will go to heat energy. The less energy that gets lost to heat, the more efficient the method is. The efficiency is determined by the energy that goes in when deforming the rubber band and the energy that gets released when going back to its original shape. If two different methods are used on the same rubber band, will the efficiency be the same?

$$Efficiency = \frac{energy\ out}{energy\ in}$$

The energy going into the rubber band is measured by the work done to deform it while the energy going out is measured by the work done by the rubber band. Work done can be calculated using the force and distance.

$$Work\ done = \ force\ \times distance\ moved\ in\ the\ direction\ of\ the\ force$$

However, as the motion of the rubber band releasing its energy is rotational after having been twisted or winded, torque will be used as an equivalent to force. In addition to this, due to the force changing direction as the rubber rotates, the distance moved in the direction of the force will be replaced by angular displacement.

$$Work\ done = torque \times angular\ displacement$$

When a certain force is applied to rotate an object attached to the end of the rubber band, the torque can be calculated by multiplying the force exerted and radius of the object. This works for measuring the work done to deform it. However, when the rubber band releases its energy, it is easier to calculate the torque by using the rotational inertia and angular acceleration. This is due to it being easier to get data of the angular acceleration when an object is in motion.

$$Torque = Force \times radius$$

$$Torque = Rotational\ Inertia \times angular\ acceleration$$

The rotational inertia is calculated by the radius and mass of an object that the rubber band does work on. This would most likely be a disk or cylinder rotating around its own axis.

$$Rotational\ inertia\ of\ a\ cylinder = \frac{1}{2} \times mass \times radius^2$$

When a rubber band is twisted the two end points are fixed, so there will a strain as the rubber band wraps around itself. However, on a winding rubber band there is a uniaxial stress applied and the energy comes from the tension created. The more energy put into the rubber band to deform it, the more energy will be stored and thereby released when untwisting or unwinding. As the rubber band deforms it gets more and more stretched for each turn, which leads to an increase in tension.

IV. Experiment 1 - measuring work done by rubber band

1. Twisting

A rubber band was cut from being a loop to a line and was hung around the perpendicular pole on a clamp stand. The two ends hanging down were attached to a rotating shaft with a cylindrical mass in a way that the rubber band was neither stretched or loose. By finding this perfect balance of distance between the pole and shaft I was able to control that no forces were pushing on the rubber band before the experiment started. On the shaft was a light gate to measure the angular acceleration.

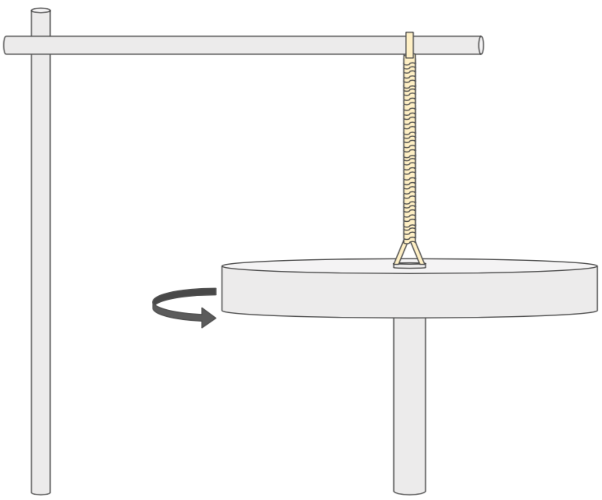

Figure 5: Illustration depicting the setup to measure the work done by a twisted rubber band

The cylinder mass was turned to twist the rubber band until it created double twists along the whole distance of the band. Then the cylinder was released to accelerate due to the force of the rubber band acting on it, and the angular acceleration of the process was being measured. Observing the untwisting of a double twisted rubber band, the turns of the cylinder ended up being 360 rad which is approximately 57 rotations. The angle was plotted against the angular acceleration to get a graph showing the energy of the rubber band untwisting on the cylinder mass, passing through the point of being untwisted and using leftover energy to twist itself again.

1.1 Results

The first thing one can see by the plotted graph is the distinct s-shape that resembles the hysteresis graph. Further one can see that the acceleration was positive until the rubber band had twisted approximately 258 rad. It becomes negative as the elastic torque is decreasing and being balanced out by the friction torque of the shaft. The elastic torque is bigger than the friction torque until the untwisting stops in the end where they are equal. The torque after 360 rad is therefore equal to the frictional torque.

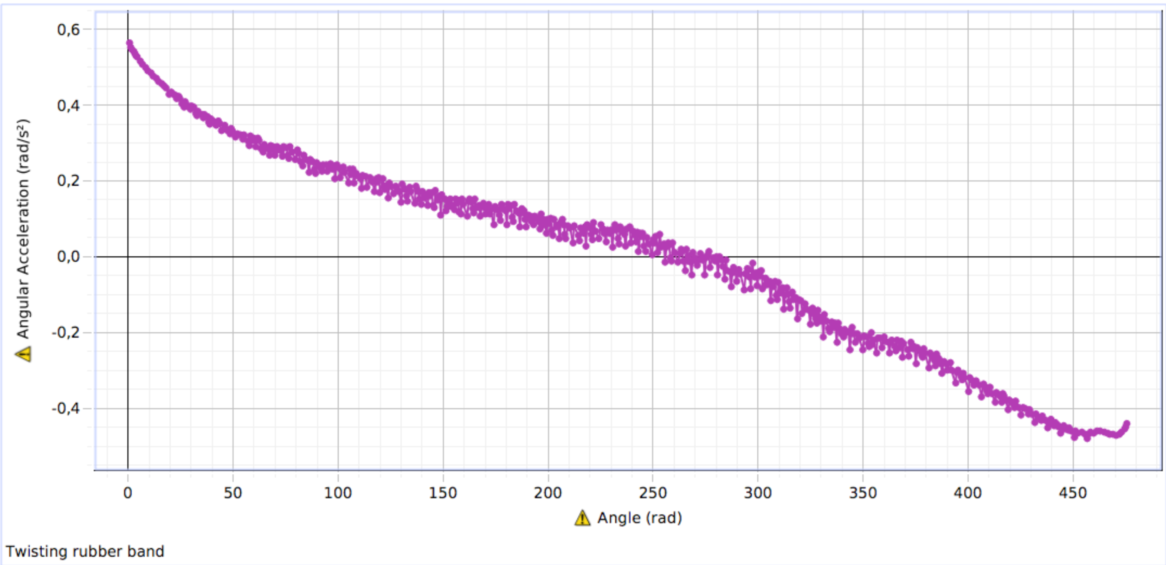

Figure 6: Graph showing the measured angular acceleration of a twisted rubber band releasing energy

If we plot the angular velocity against the angle, we can see that the maximum velocity was $10.33\ rad/s$ which after approximately 250 rad. Comparing with the acceleration-angle graph we can see that this is at the point where the friction torque started balancing out the elastic torque.

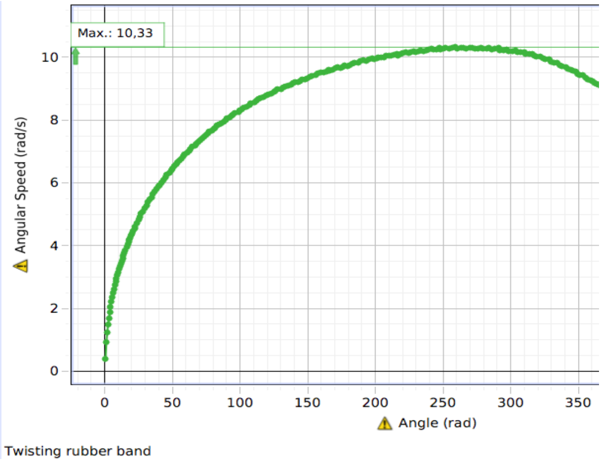

Figure 7: Graph showing max velocity for twisted rubber band

Looking at the graph at a closer distance one can see that the graph is not completely smoothly accelerating at the beginning, but rather having a brake at an increment of 2π. Considering the observations made, this was when each double knot untwisted causing an uneven behaviour of the velocity after each rotation.

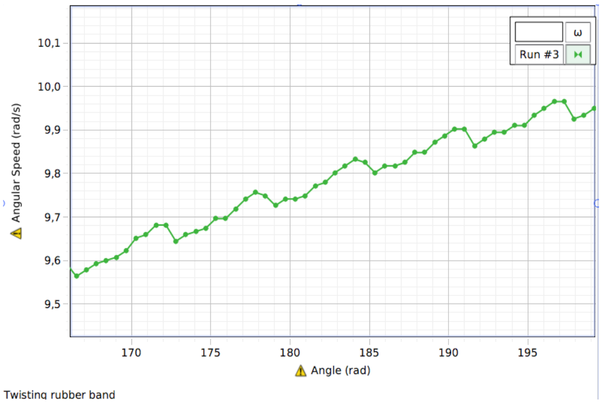

Figure 8: Graph showing the uneven speed due to double twists

1.2 Calculating the work done

To find the energy stored in the rubber, one needs to figure out the work done by it. The work done on an accelerating body is calculated by the area under a graph plotting the torque against the angular displacement. On figure 6, the data registered is from the work done on the cylinder and not by the rubber band. This is different due to the frictional torque balancing out the elastic torque. We will therefore first have to figure out the total torque and then subtract the frictional torque.

$$Work\ done\ by\ rubber\ band = elastic\ torque \times angular\ displacement$$

The torque(τ) is found using the rotational inertia(I) of the cylinder and the angular acceleration(α).

$$Work\ done = \tau \times angular\ displacement$$

$$\tau = I \times \alpha$$

$$I = \frac{1}{2}mr^2 = \frac{1}{2} \times 1.383\ kg \times (0.114\ m)^2 = 0.008986734\ kgm^2$$

$$I = 8.99 \times 10^{-3}\ kgm^2 \pm 0.09 \times 10^{-3}$$

To find the total torque the angular acceleration was measured at an increment of 20 rad. This data is extracted by taking the average points from a smoothed version of the graph in figure 6. The total torque is then calculated multiplying the rotational inertia and the angular acceleration, which, as stated earlier, will give the torque for the work done on the cylinder.

To figure out the elastic torque(τ_e) the friction torque(τ_f) was subtracted from the total torque(τ_t). The friction torque was calculated to be 2.07×10^{-3}Nm

$$\tau_e = \tau_t - \tau_f$$

$$\tau_e = \tau_t - 2.07 \times 10^{-3}$$

Angle	Angular acceleration (rad/s) ± 0.03	Total torque (Nm)	Elastic torque(Nm) ± 0.0003
0	0.56	0.00503	0.0071
20	0.43	0.00387	0.0059
40	0.36	0.00324	0.0053
60	0.31	0.00279	0.0049
80	0.25	0.00225	0.0043
100	0.23	0.00207	0.0041
120	0.19	0.00171	0.0038
140	0.16	0.00144	0.0035
160	0.14	0.00126	0.0033
180	0.11	0.00099	0.0031
200	0.08	0.00072	0.0028
220	0.06	0.00054	0.0026
240	0.05	0.00045	0.0025
260	0.02	0.00018	0.0022
280	-0.02	-0.00018	0.0019
300	-0.06	-0.00054	0.0015
320	-0.13	-0.00117	0.0009
340	-0.2	-0.00180	0.0003
360	-0.23	-0.00207	0.0000

Figure 9: Table demonstrating the calculation of elastic torque on a twisted rubber band

Then a graph was created plotting the elastic torque against the angular displacement. Work done by the rubber band is the area under the graph.

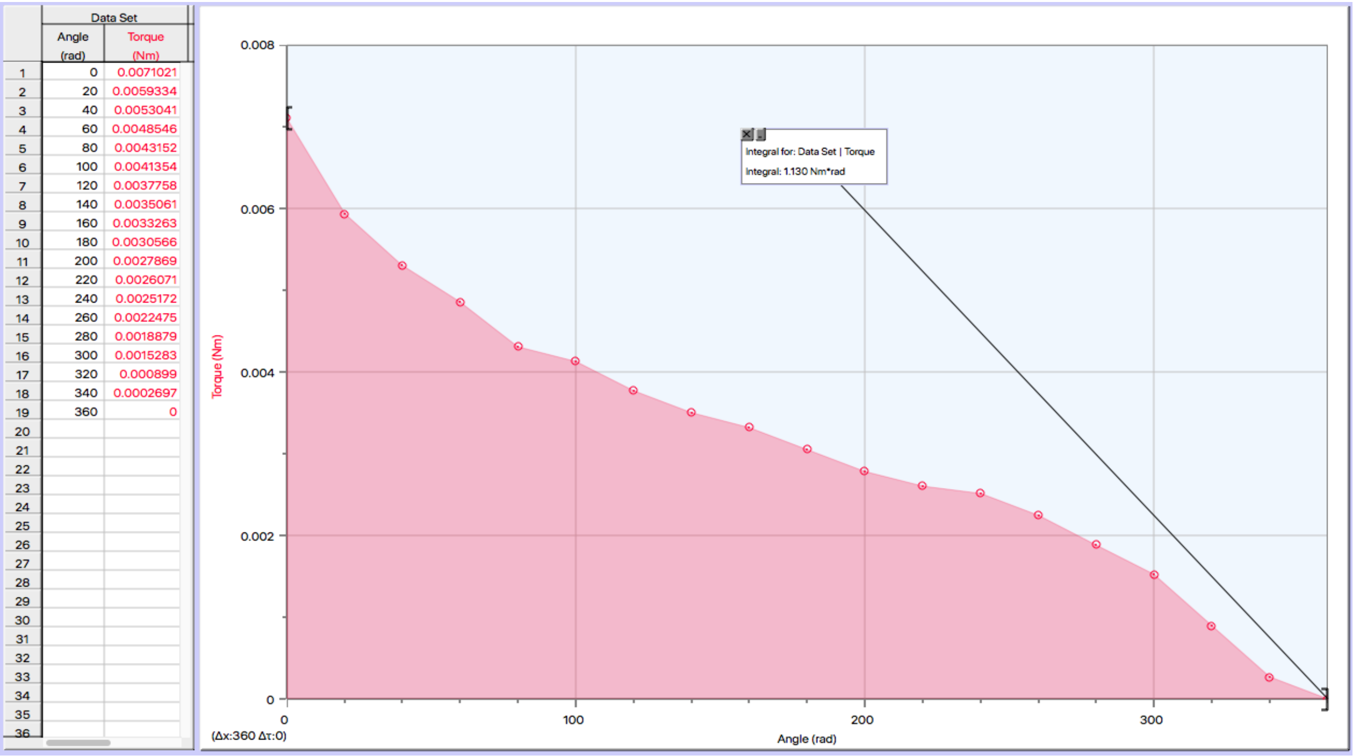

Figure 10: Graph showing energy stored in twisted rubber band

From looking at the graph one can see that the energy stored by twisting this rubber band 360 rad stores 1.13 Joules.

2. Winding

The same clamp-stand and cylindrical mass on a shaft was used in the experiment for the winding, but the position of the rubber band changed to go horizontal instead of vertical, attaching one end to the shaft and the other to a stand.

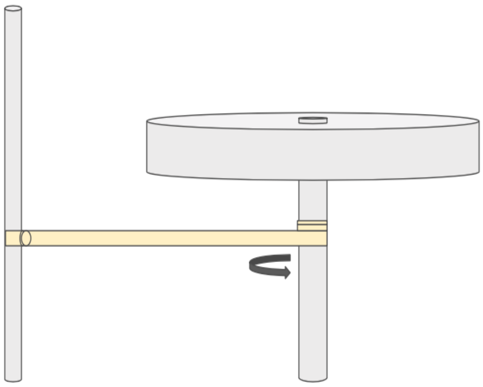

Figure 11: Illustration depicting the set-up for measuring work done by a winded rubber band

The rubber band was winded by turning the shaft making the band wrap itself up around it. By observation one can see that this method of storing energy in the rubber band takes fewer turns before the tension becomes strong. After approximately 6 turns, the cylinder was released to let the rubber band unwind. As with the twisted rubber band angular acceleration was plotted against the angular displacement.

2.2 Results

One can see that the observation of this method taking fewer turns was accurate as it only reached a rotation of 37 rad compared to 360 rad for the twisting.

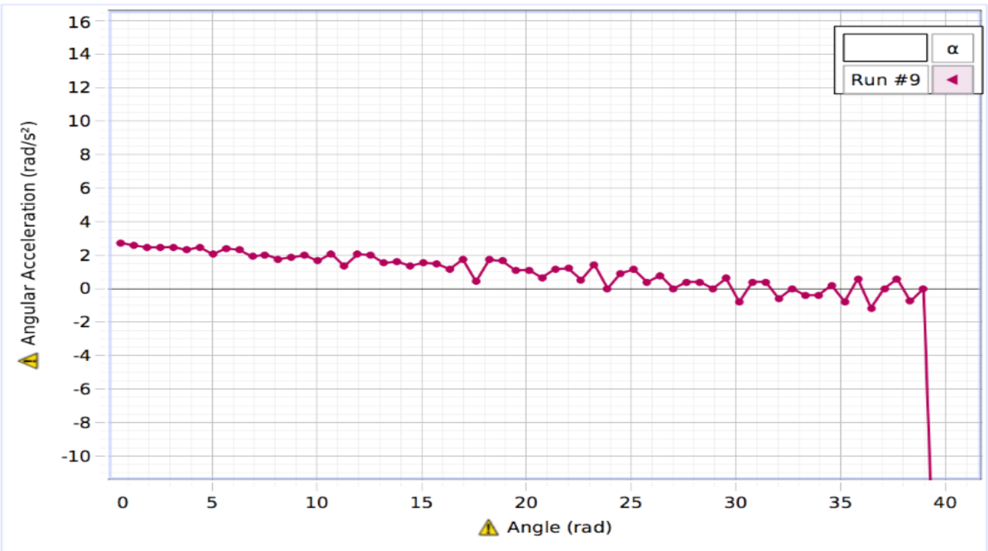

Figure 12: Graph showing the measured angular acceleration of winded a rubber band releasing energy

To further investigate velocity was plotted against angle to get a graph that showed the maximum velocity to be 9.27rad/s. This happened after approximately 30 rad. After that the friction torque started to balance out the acceleration.

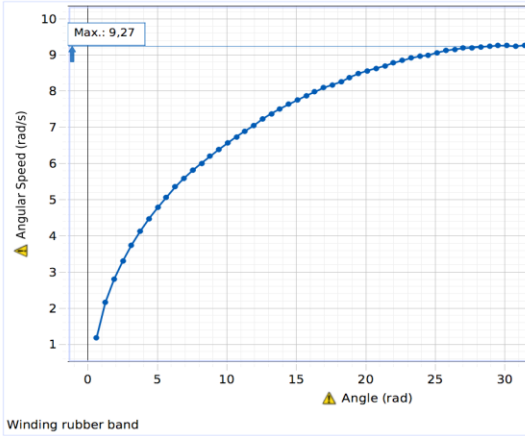

Figure 13: Graph showing max velocity for winded rubber band

To find the total torque, measurements of the angular acceleration were taken at an increment of 2 rad. These measurements were multiplied by the rotational inertia to calculate the absolute torque and the elastic torque. I estimate that these won't be very different from each other as the torque was much stronger and as we see on the graph the friction torque balanced much later compared to with the twisted rubber.

2.2 Calculating the work done

Angle	Angular acceleration (rad/s) ± 0.03	Total torque (Nm)	Elastic torque (Nm) ± 0.0004
0	2.65	0.0238235	0.0273
2	2.51	0.0225649	0.0261
4	2.43	0.0218457	0.0254
6	2.28	0.0204972	0.0240
8	1.84	0.0165416	0.0200
10	1.92	0.0172608	0.0208
12	1.84	0.0165416	0.0200
14	1.46	0.0131254	0.0166
16	1.44	0.0129456	0.0165
18	1.18	0.0106082	0.0141
20	1.06	0.0095294	0.0130
22	0.98	0.0088102	0.0123
24	0.63	0.0056637	0.0092
26	0.56	0.0050344	0.0085
28	0.27	0.0024273	0.0059
30	-0.09	-0.0008091	0.0027
32	-0.24	-0.0021576	0.0013
34	-0.25	-0.0022475	0.0013
36	-0.38	-0.0034162	0.0001
37	-0.39	-0.0035061	0.0000

Figur

Here we can see that the energy stored in the rubber band when winded 37 rad was 0.503 Joules

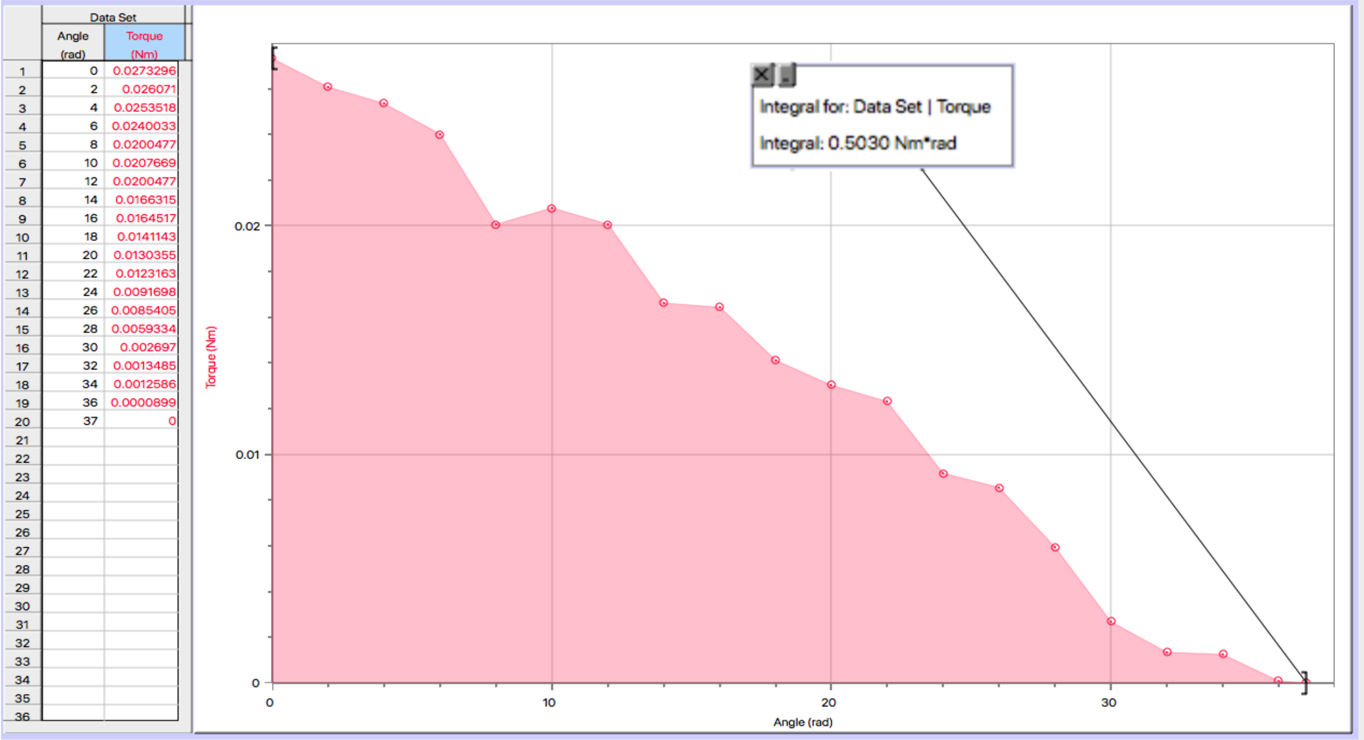

Figure 15: Graph showing the energy stored in a winded rubber band

3. Evaluation of results to experiment 1

Comparing the release of energy in the two different methods, we can clearly see some differences due to the contrasting deformations done to store the energy. Firstly, looking at the velocity one can see that the twisting method reached a velocity of 10.33 rad/s while the winding method only reached 9.27rad/s. The acceleration on the other hand was much higher in the beginning for the winding rubber band compared to the twisted rubber band. The winding method started with an acceleration of $2.65\ rad/s^2$while the twisting started with $0.56\ rad/s^2$. In addition, this leads to most of the acceleration for the winded rubber band to be positive due to the elastic torque being much bigger than the friction torque, making it negligible. From observation one can notice that the winding had a much bigger force before releasing but took fewer turns to unwind. This, in conclusion, led to a much quicker release of energy for the winded rubber band compared to the twisted rubber band.

The energy stored in the rubber band for the twisting was 1.13 Joules and for the winding 0.503 Joules. This difference was most likely due to the difference in work done to deform the rubber band. One can observe in the graphs of work done when releasing energy of the twisted rubber band that it looks similar to the hysteresis graph whereas the graph for winding looks more linear. This might be due to the lack of work done to deform the winded rubber band. To make the difference in energy released comparable, the work done to deform the rubber bands will be measured for the two methods.

V. Experiment 2 - measuring work done to deform the rubber band

1. Twisting

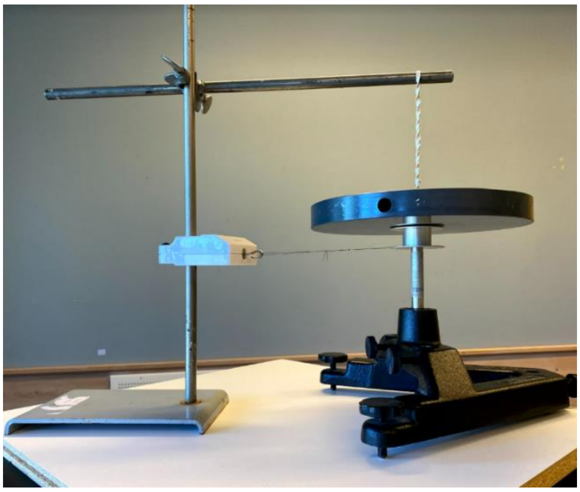

Figure 16: Set-up for measuring work done to twist the rubber band

To measure the work done to deform the rubber band the force was measured at different stages of twisting. To prepare for this method the same set up from experiment 1 was made without the light gate but rather a short string attached to the shaft. The force was measured by attaching the short string to a force sensor. This was being done at an increment of 5 rotations until 60 rotations, which is approximately 360 rad where the rubber band has double twists. This process was done 5 times to get an average value that would be more accurate. By using the collected data of the force, a graph of torque against angular displacement was made. The area under that graph was the work done to twist it.

1.1 Processed data

Rotations	Angle (rad) ± 0.1	Force (N) ± 0.01					Average force	Torque (Nm) ± 0.0003
		1	2	3	4	5		
10	62.8	0.08	0.06	0.08	0.08	0.07	0.074	0.0019
15	94.2	0.11	0.11	0.11	0.11	0.1	0.108	0.0028
20	125.7	0.12	0.11	0.12	0.11	0.11	0.114	0.0030
25	157.1	0.12	0.12	0.12	0.12	0.13	0.122	0.0032
30	188.5	0.15	0.14	0.14	0.14	0.13	0.14	0.0036
35	219.9	0.16	0.15	0.15	0.15	0.15	0.152	0.0040
40	251.3	0.16	0.17	0.15	0.15	0.15	0.156	0.0041
45	282.7	0.16	0.16	0.16	0.15	0.16	0.158	0.0041
50	314.2	0.18	0.18	0.18	0.17	0.17	0.176	0.0046
55	345.6	0.21	0.21	0.17	0.18	0.19	0.192	0.0050
60	377.0	0.21	0.22	0.18	0.2	0.21	0.204	0.0053

Figure 17: Table of measured force per angle for twisting a rubber band

I figured out that the work done for twisting a rubber band 360 rad is 1.10 Joules

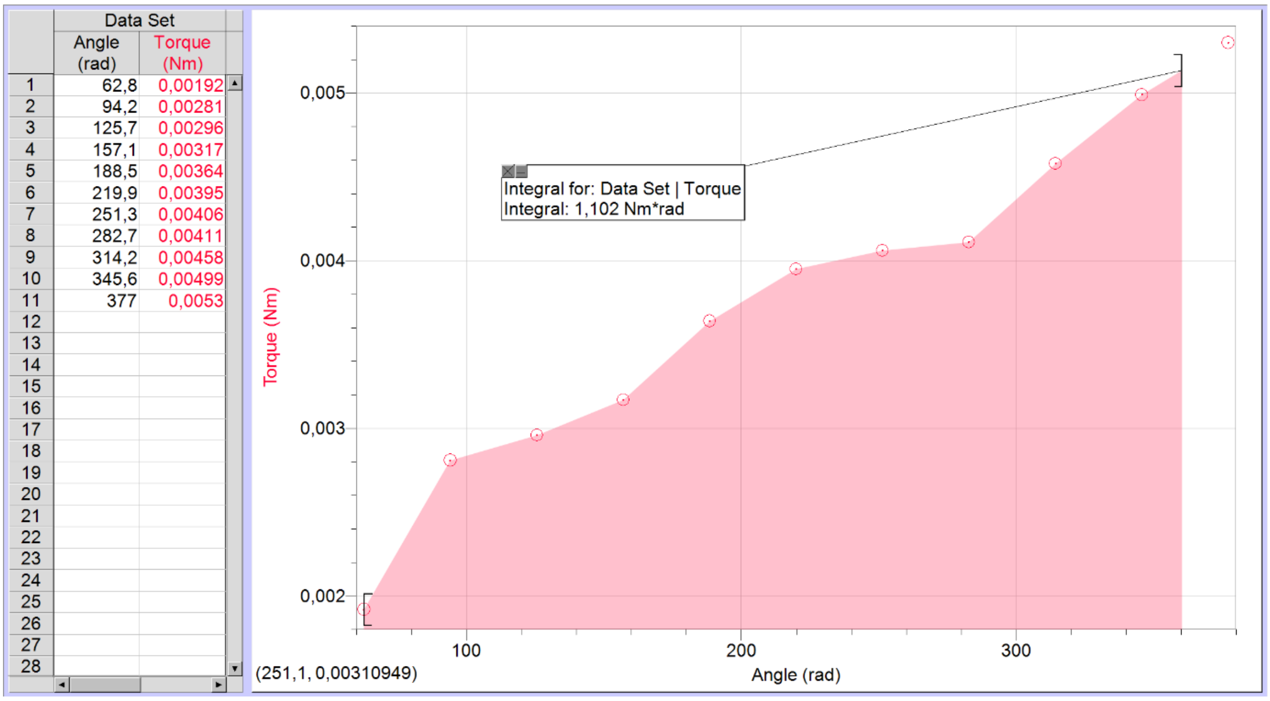

Figure 18: Graph showing work done to twist a rubber band by 360 rad

2. Winding

The setup of the rubber band was like the one from the winding experiment 1. The force was again measured by using a string to the shaft and a force sensor. This was done at an increment of 1 rotation as there were far less rotations for this method. Then the measuring process was repeated 5 times to find an average force and the data was used to plot torque against angular distance.

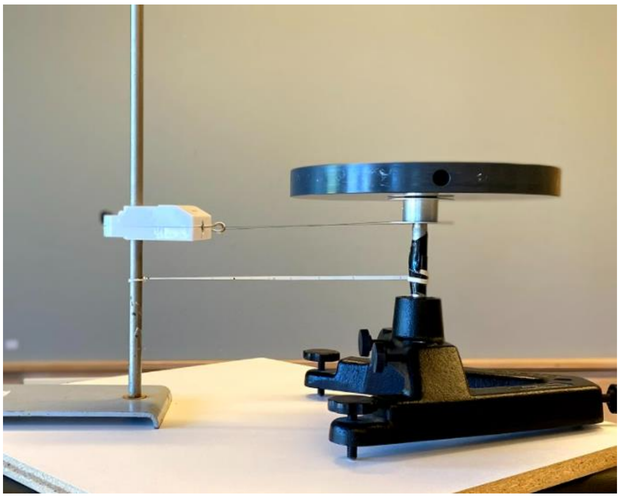

Figure 19: Set-up for measuring the work done to wind the rubber band

2.1 Processed data

Rotations	Angle (rad) ± 0.1	Force (N) ± 0.06					Average force	Torque (Nm) ± 0.002
		1	2	3	4	5		
1	6.3	0.35	0.3	0.29	0.31	0.27	0.304	0.008
2	12.6	0.59	0.57	0.55	0.56	0.54	0.562	0.015
3	18.8	0.73	0.74	0.73	0.72	0.67	0.718	0.019
4	25.1	0.84	0.88	0.85	0.85	0.82	0.848	0.022
5	31.4	0.92	1.01	0.99	0.99	0.96	0.974	0.025
6	37.7	1.09	1.2	1.2	1.18	1.14	1.162	0.030
7	44.0	1.27	1.43	1.45	1.41	1.4	1.392	0.036
8	50.3	1.53	1.86	1.76	1.79	1.76	1.74	0.045

Figure 20: Table of measured force per angle for winding a rubber band

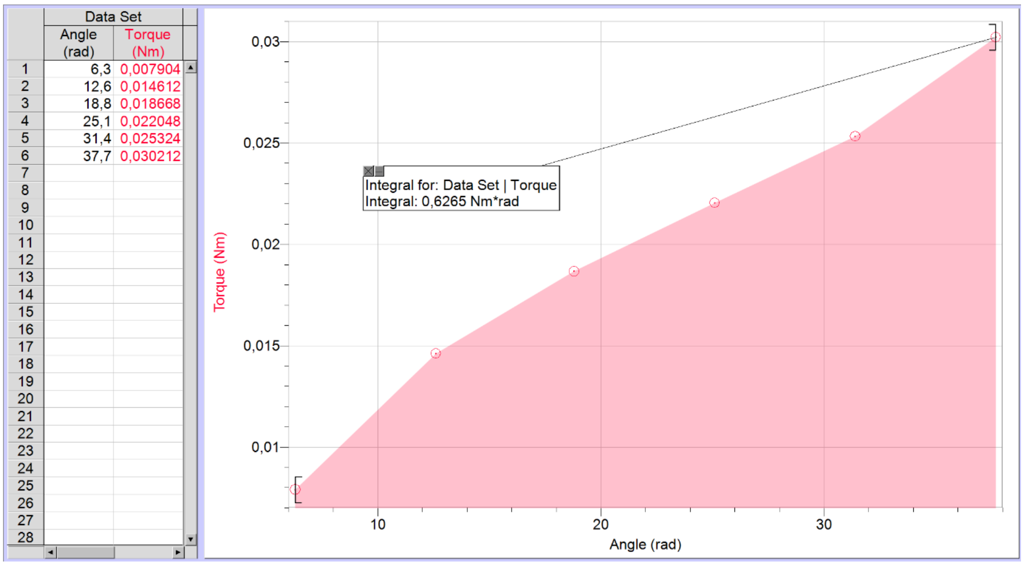

Figure 21:Graph showing work done to wind a rubber band by 37 rad

The work done for winding 37 rad was 0.63 Joules

VI. Comparing results

After having collected data of the energy stored in the rubber band and the work done to deform it, we can start comparing the two methods. By looking at the results below we can see that the work done to deform the two rubber bands are different. This means that since the same amount of energy hasn't been used to deform them and the energy stored will therefore not be comparable. To be able to make a good comparison, a new experiment was done where the more energy is being used for winding the rubber band.

	Experiment 1 (work done by rubber band releasing energy)	Experiment 2 (work done to deform rubber band)
Twisting	1.13 Joules	1.10 Joules
Winding	0.50 Joules	0.63 Joules

VII. Redoing of experiment

As we want to figure out the most efficient method for storing the highest amount of energy it was decided that the experiments for the winding rubber bands should be repeated by doing the same amount of work that had been done to the twisted rubber band. To do so we had to figure out the number of rotations it took to do the work of 1.10 Joules to wind the rubber band as that's the amount it took to twist it by 360 rad. The force was measured and plotted against the angle. After having used the energy of approximately 1.10 Joules the rubber band had been rotated by 50 rad.

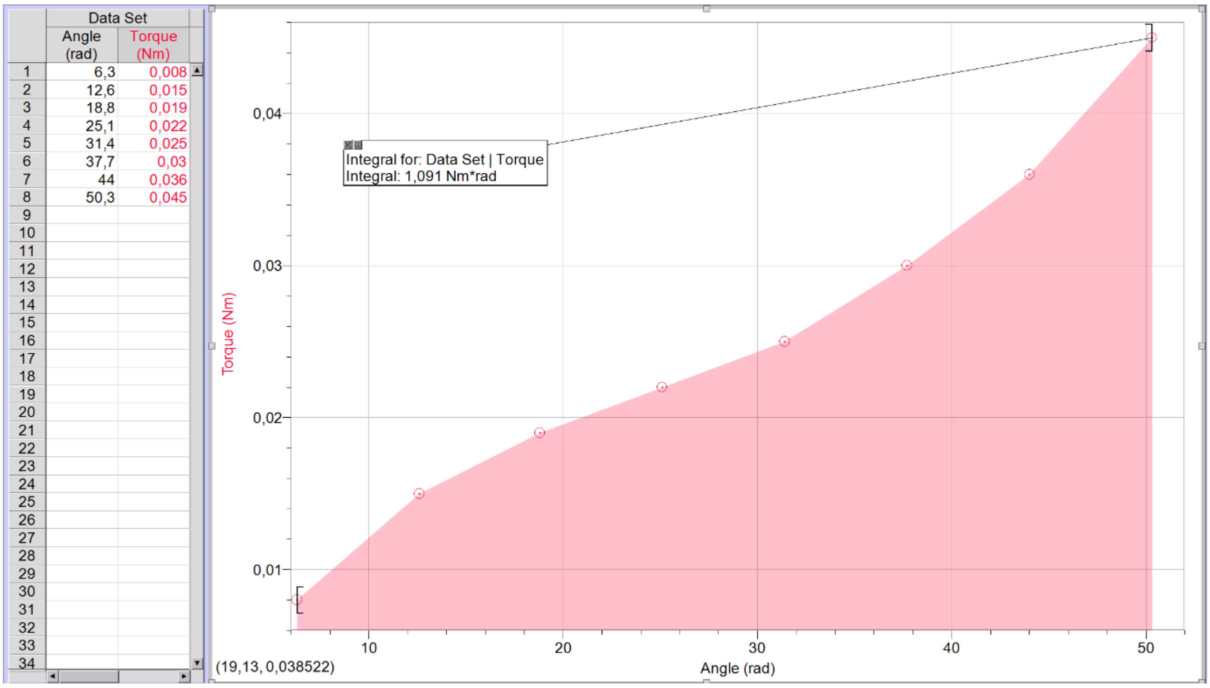

Figure 22: Work done to wind a rubber band by 50 rad

In figure 22 we can see that the energy used to wind the rubber band 50 rad is approximately 1.10 Joules. This information was used to figure out how much energy could be stored in the rubber band by doing the same amount of work to deform it as to the twisted rubber band. To do so experiment 1 was repeated for the winded rubber band twisting it by 50 rad. When released the angular acceleration and angle were measured to further calculate the elastic torque. The torque was plotted against the angular distance to get a graph where the area under was the work done by the rubber band. In the illustration under there is a graph of work done by a rubber band that was winded around 58 rad. By only calculating the area under the graph for the last 50 rad we get the work done by a rubber band that has been winded using 1.10 Joules.

The winded rubber band stored 0.97 Joules.

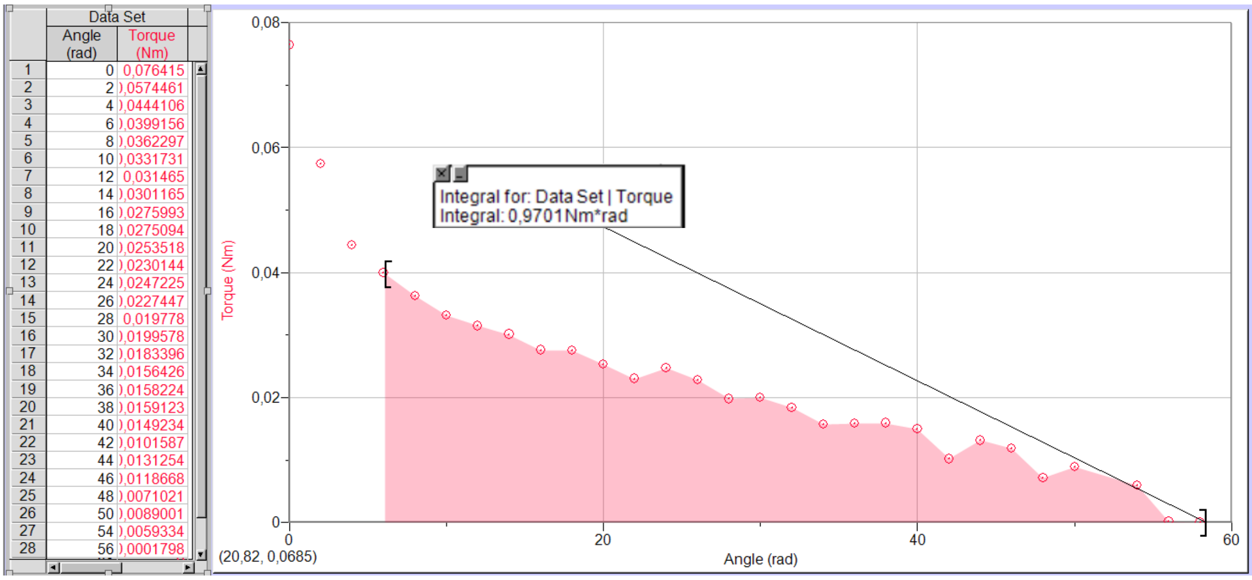

Figure 23: Graph showing Energy stored in a rubber band winded by 50 rad

By doing this experiment of winding the rubber band by approximately 58 rad it reached a maximum velocity of 15.99 rad/s

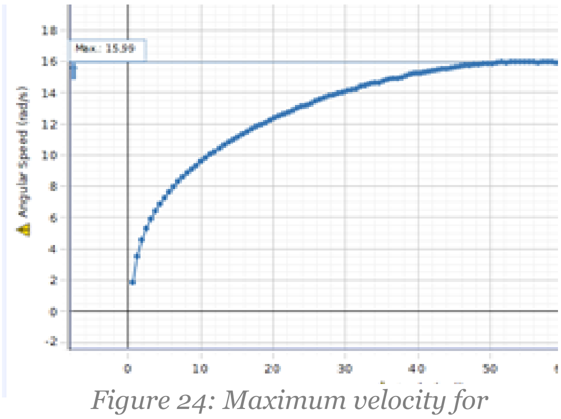

Figure 24: Maximum velocity for a winded rubber band

VIII. Conclusion

Now that there is information for the two different methods where the same amount of work is being done to store energy, it is possible to compare them equally. I have measured the energy stored and released in the rubber band when approximately 1.10 Joules have been used to deform it for the two different methods. By that I calculated that the twisted rubber band released 1.13 Joules while the winded rubber band only released 0.97 Joules. By knowing this, the efficiency for the two methods can be calculated to show that the twisting method is more efficient than the winding method.

$$Efficiency\ twisting = \frac{1.13}{1.10} = 1.03$$

$$Efficiency\ winding = \frac{0.97}{1.10} = 0.88$$

Nevertheless, during the release of energy in the winded rubber band it reached a maximum velocity of 16.0 rad/s compared to 10.3 rad/s for the twisted rubber band. This might have a connection to the winded rubber band releasing its energy in a very short time in contrast to the twisted band. This was due to the difference starting torque for two methods. Looking at the data we can see that while the twisted method only had 0.5 Nm before it released its energy, the winded method had a torque of 4.5 Nm.

IX. Evaluation

It was concluded that the energy released in the twisted rubber band was more than the energy used to deform it. This is not possible according to the law of conservation of energy and we assume that the difference was due to an error. One reason for this error might have been caused by the difference of static and dynamic measuring. When deforming the rubber band, the force was measured at different increments whereas when released the acceleration was measured. Despite force and acceleration being proportional, external factors such as friction or air resistance might have affected the measurements. Another reason might be due to uncertainties. If one plots the maximum values for the deformation graph and minimum for the graph of released energy, the values are possible according to the theory. Therefore, it can be assumed that the values are simply very close to each other, which means that not a lot of kinetic energy was transferred to heat energy.

The values calculated from the energy in the winded rubber band seem reasonable, but the way they were measured in experiment 2 can be questioned. This is because the measurements were taken at an increment of one rotation of the cylinder, assuming that the force was constant between each measurement. This is not precise as the force is increased as the band was winded, and the measurements could have been taken at smaller increments to get more accurate values.

According to the theory there should not have been any difference in the energy released for the two methods due to the same amount of work being done to deform the band. One reason to explain this difference is due to a random error caused by the rapid release of energy in the wound rubber band. Commenting on the theory another reason might have been that more of the energy used to deform the rubber band was transferred to heat energy for winding than twisting. As noticed, the winded rubber band has a much stronger force before releasing its energy due to the band having different amounts of tensions throughout it. This happens as the band becomes more and more stretched with each rotation leading to different thicknesses around the shaft (see figure 23). The band then ends up having a much bigger force for the winded method compared to the twisting that has a consistent stretch throughout the band. Acknowledging the theory, it can be concluded that the area between stretching and releasing on the hysteresis graph is bigger for the winded rubber band than the twisted band which leads to the twisting method to be more efficient.

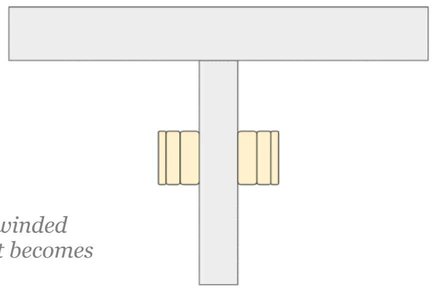

Figure 25: Cross section of winded rubber band showing that it becomes thinner by each rotation

X. Discussion

If one is to consider a rubber band to be used to power a vehicle, using the twisting method does not only make it more efficient, but due to the less of a force it has when being deformed, it would also be able to hold more energy without breaking. The lack of force also makes it easier to deform it when twisting compared to winding. On the other hand, the winded method has a much higher velocity than the twisted. However, this is due to the rapid release of energy that makes the movement very uncontrollable and messy, whereas the twisted method releases the energy in a steady and controlled way. It can therefore be concluded that the twisted method is not only more efficient, but also more appropriate to store and release energy.

Knowing this it can be discussed how this information can further be applied practically to store and release energy. For example, if twisted rubber bands could be used as a substitute for gasoline in vehicles. By calculating the energy per kg for the twisted rubber band it is discovered that it has a specific energy of $1\ 083\ Joules/kg$. This is a low value compared to an experiment done by Rhett Allain where he calculated a rubber band to have a specific energy of $6\ 605\ Joules/kg$ (Allain, 2018) and could be increased by changing variables such as length, width, or thickness of the rubber band. Nevertheless, both values are very low compared to gasoline which has an energy density of $45\ 000\ 000\ Joules/kg$. Therefore, a twisted rubber band might not be a comparable material to gasoline, however considering its efficiency it is still a great method for storing and releasing energy for smaller vehicles or other objects. As seen in the difference of specific energy that was found in this experiment and Allain's experiment, a further investigation could be about what proportions between length, width, and thickness that would be the best to store the most amount of energy.

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