IBDP Mathematics Analysis and Approaches SL

Internal Assessment:

Finding the Volume of an Avocado

19 Pages

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Contents

1 Introduction

When we were introduced to the Mathematics Internal Assessment, I was excited at the prospect of finally getting a chance to apply maths to the real world past basic geometry. I had many ideas in the way of finding the volume of an object, but I had to find one that I could use the curriculum's mathematics to solve. We had already studied derivation in class and I was aware that integration would be useful to solve for the volume of anything. After a discussion with my teacher I decided to learn it on my own to complete the goal of finding the volume of something.

The idea to find the volume of an avocado came to me while making avocado on toast. I wanted to know how much an avocado fruit one might be able to use when making their morning toast. This is additionally relevant outside of my morning toast, in areas such as avocado growth in orchards. Finding the weight of an avocado before it has been picked based solely on measurements is important in calculating the total yields for farmers (Lahav & Zamet, 1971), and understanding the maturity of an avocado based on it's volume and weight can improve farmers efficiency. I quickly realised that to accurately find the volume of an avocado I would have to exclude the volume of it's pit. After considering my options, I decided that as I would be learning integration on my own and attempting to model the already obscure shape, to exclude the pit as a consideration and find the total volume of the avocado. The aim of this exploration is to mathematically find the volume of an avocado including it's pit.

2 Experimentally Testing the Volume of the Avocado

I started my exploration by finding a base point with which to compare my calculations. I decided to use the Archimedian method to find the volume. To do this I filled a large jug with water until the avocado could be fully submerged. It must be noted that the measurement markings on this jug were not particularly accurate, however it was the only jug available that was large enough. After submerging the avocado in the water,

the change in volume I observed was 350ml which is equal to 350cm^3.

3 Modelling the Avocado

When considering how I would determine the volume of the fruit, I initially approached this using simple methods such as using existing shapes that could possibly model my avocado. I also thought about shapes such as ellipses and ovoids that were similar to the shape of an avocado. Something I knew I needed to consider was how accurate all these methods would be in the end. I decided to start with something simpler and work my way up to more refined models if I needed to do so.

3.1 Two Cylinders Approach

I started by trying to model the avocado using a set of cylinders in an attempt to simplify the situation. I traced the avocado's largest and smallest section to the best of my ability, then measured the two diameters using a 30cm ruler. Using this method I found the radius of the two cylinders I would be modelling: 1.75cm for the smaller cylinder and 3.8cm for the bigger one. I chose these radii based on the radii of the largest and smallest traceable parts of my avocado. I used the *Geogebra* 3D Graphics tool[1] to create them as can be seen in Figure 1.

(a) Option A   (b) Option B

Figure 1: Two cylinder model of volume with radii of 3.8cm and 1.75cm

When making the cylinders I made sure to have a maximum height of 12cm to show the measured length of the avocado I found by tracing the side view. Using the same radii

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[1]https://www.geogebra.org/

I made several more models using different heights of the lower cylinder. An example can be seen in Figure 1b. After I had modelled the first set of cylinders I attempted to find the volume of these using the following formulae:

$$V = \pi r^2 h$$

$$V_{total} = \pi(3.8cm)^2 6.6cm + \pi(1.75cm)^2 5.3cm$$

$$V_{total} = 299.4cm^3 + 50.99cm^3$$

$$V_{total} = 350.4cm^3$$

I used the same method to find the total volume for figure 1b and found it to be 401cm^3.

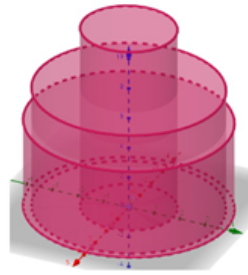## 3.2 Three Cylinder Approach

Figure 2: Three Cylinders of radii 3.8cm, 3.5cm and 1.75cm

I next tried to model the avocado using three cylinders with radii of of 3.8cm, 3.5cm and 1.75cm. The same same total height as the previous model of 12cm was used. In the example in Figure 2 the largest cylinder had a height of 5.78cm, the second largest a height of 2.17cm and the smallest a height of 4.05cm An example of this can be seen in Figure 2. I found the volume of this model using the same formula as in Section 3.1 in the following manner:

$$V = \pi r^2 h$$

$$V_{total} = \pi(3.8cm)^2 5.78cm + \pi(3.5cm)^2 2.17cm + \pi(1.75cm)^2 4.05cm$$

$$V_{total} = 262.20cm^3 + 83.51cm^3 + 38.97cm^3$$

$$V_{total} = 384.68cm^3$$
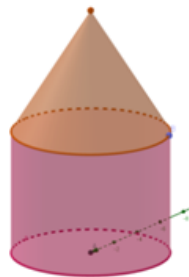
3.3 Cylinder and Cone Approach

Figure 3: Cone on top of a cylinder. The radius of both shapes is 3.8cm

In my next attempt to model the avocado, I used a cylinder with a cone on top to try to mimic the shape of the avocado, this is shown in Figure 3. I used the following method to find the volume:

$$V_{cone} = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(3.8cm)^2 \times 6cm = 90.73cm^3$$

$$V_{cylinder} = \pi r^2 h = \pi(3.8cm)^2 \times 6cm = 272.19cm^3$$

$$V_{total} = 90.73cm^3 + 272.19cm^3$$

$$V_{total} = 362.92cm^3$$

3.4 Sphere and Cone Approach

My final attempt to model the avocado using only existing 3D shapes was to use a cone on top of half a sphere (hemisphere) as depicted in Figure 4. Both the cylinder and the cone had a radius of 3.8cm and added up to a total height of 12cm. I found the volume by doing the following:

$$V_{hemisphere} = \frac{\frac{4}{3}\pi r^3}{2} = \frac{\frac{4}{3}\pi \times (3.8cm)^3}{2} = 114.92cm^3$$

$$V_{cone} = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(3.8cm)^2 \times (12cm - 3.63cm) = 126.57cm^3$$

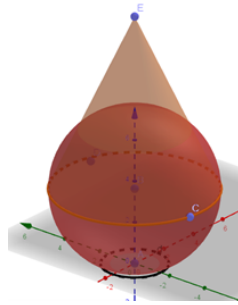$$V_{total} = V_{hemisphere} + V_{cone} = 241.49cm^3$$

Figure 4: Cone on top of a hemisphere where both radii are 3.8cm

3.5 Comparison of Shapes

Table 1: Shape combination *vs.* Volume

Shape Combination	Values	Volume
2 Cylinders	$r_1 = 3.8cm$ $r_2 = 1.75cm$	$350.4cm^3$
2 Cylinders	$r_1 = 3.8cm$ $r_2 = 1.75cm$	$401cm^3$
3 Cylinders	$r_1 = 3.8cm$ $r_2 = 3.5cm$ $r_3 = 1.75cm$	$384.68cm^3$
Cylinder and Cone	$r = 3.8cm$ $h = 6cm$	$362.92cm^3$
Hemisphere and Cone	$r = 3.8cm$ $h = 8.37cm$	$241.49cm^3$

Thus far the average volume of all models I had made was 308.17cm^3. See a summary of all calculated volumes in Table 1. I chose to only use two decimal places after each of my answers because that is the smallest increment of a centimetre. This simplified the equations, but I have noted that there may be a small uncertainty due to rounding off the numbers. I have also noted that there is significant uncertainty in the volume of the avocado using these models. There are significant volumes that are added or

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removed from an avocado's shape using these models which would not exist in the real avocado's volume. These volumes could include a larger bottom half and a cone like top half with a flat top. These models also do not account for the dips and curves of the avocado.

3.6   Revised Ellipse Model

I then realised that I needed a more accurate model of the avocado. To start creating a more accurate model, I used a set of digital calipers to make more accurate measurements. In this I found that the length of my avocado was actually 11.0cm (Figure 5), rather than the 12cm I had originally measured.

Figure 5: Digital calipers measuring 110.74mm $\approx$ 11.0cm

Using this and other measurements I was able to continue in making my model. I first thought of using an ellipse as it is the simplest shape that could model the side of an avocado. I found an equation for an ellipse centred at the origin with the X-axis as the major axis. As defined below:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Where $a > b$, and a is the half length of the major axis (X-axis) and b is similarly half the length of the minor axis (Y-axis) (Abramson, 2021).

To model this on a graph I had to transform it into a function for calculation.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$
$$\frac{y^2}{b^2} = 1 - \frac{x^2}{a^2}$$
$$y = \sqrt{(\frac{1-x^2}{a^2}) \times b^2}$$

I modelled this function with an image of the avocado's side in the background for comparison. I chose to use the smaller more even side of the avocado to model because it would simplify the situation. Since I already knew the length of the avocado on the X-axis to be 11cm, I could immediately input $a = \frac{11}{2} = 5.5$. I was however unsure about what to input for *b*. I chose to put a slider into *Geogebra* to adjust *b* to different values that may change the function to fit more appropriately to the shape of my avocado. I chose to use 3 different values for variable *b*: 2.3, 3.7, 4.5. The results of this can be seen in Figures 6 (a-c).

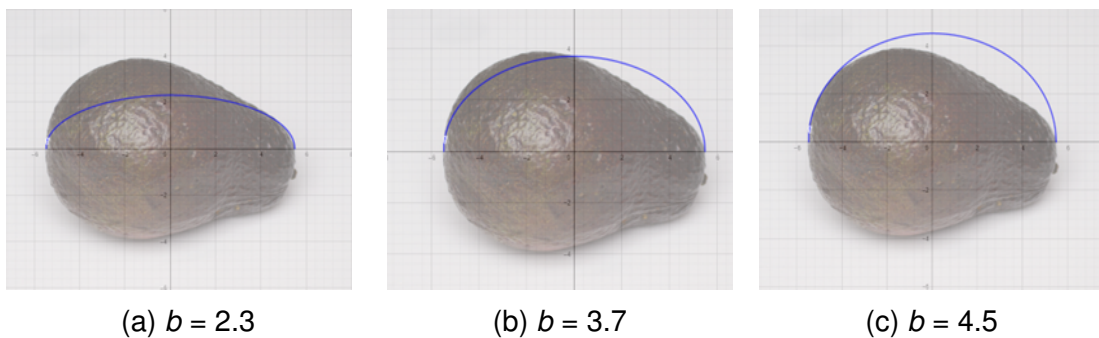(a) *b* = 2.3   (b) *b* = 3.7   (c) *b* = 4.5

Figure 6: Ellipse functions with varying minor axis

It can be seen that none of these functions fit the avocado adequately. I knew then that I needed a new function similar to this ellipse, but that would provide a better fit.

3.7 Revised Ovoid Model

After doing some research I came across an addition to the ellipse function that can model an egg (ovoid) shape, which better matches that of the avocado. This adjustment

makes one side of the shape larger and narrows it down on the other side (Köller, 2000). This function, named $t(x)$, is multiplied by y^2 in the original ellipse function and is stated as the following:

$$t(x) = \frac{(1 + kx)}{(1 - kx)}$$

This is multiplied by the y variable as the part of the ellipse that it changes is on the y-axis rather than the x-axis. I then had to create the new function to input into *GeoGebra*.

$$\frac{x^2}{a^2} + \frac{y^2\left(\frac{(1+kx)}{(1-kx)}\right)}{b^2} = 1$$

$$\frac{y^2\left(\frac{(1+kx)}{(1-kx)}\right)}{b^2} = 1 - \frac{x^2}{a^2}$$

$$y^2\frac{(1 + kx)}{(1 - kx)} = \frac{1 - x^2}{a^2} \times b^2$$

$$y = \sqrt{\frac{1 - \frac{x^2}{a^2} \times b^2}{\frac{1+kx}{1-kx}}}$$

I put in a slider for *k* and *b* and kept *a* consistent (at $a = \frac{11}{2} = 5.5$) as the length of the avocado had not changed. I quickly found that the only value for *k* that would give the appropriate egg shape was 0.1. I then only changed the value of *b*. I used 2 different values for *b*: 2.9, 3.8, as can be seen in Figure 7(a-b).
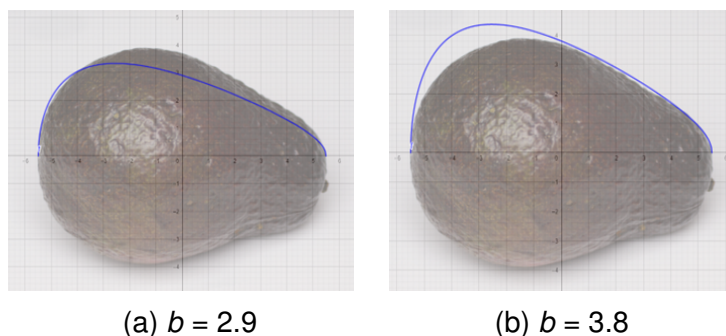

(a) *b* = 2.9   (b) *b* = 3.8

Figure 7: Egg functions

although closer, these functions again did not fit the shape of the avocado adequately.

3.8   Regression Line Model

My final attempt to model the avocado was to try use a regression line that would model the side of the avocado. To do this placed several points around the edge of the avocado in *Geogebra*. I then inputted those points into a *Geogebra* spreadsheet (see points used in Table 2) and used the *Two Variable Regression Analysis* tool[2] to create a regression line, see Figure 8. I then chose the polynomial option and increased the highest power until I found a shape that fitted the points well.

Table 2: X and Y values used in the *Two Variable Regression Analysis* on *Geogebra*

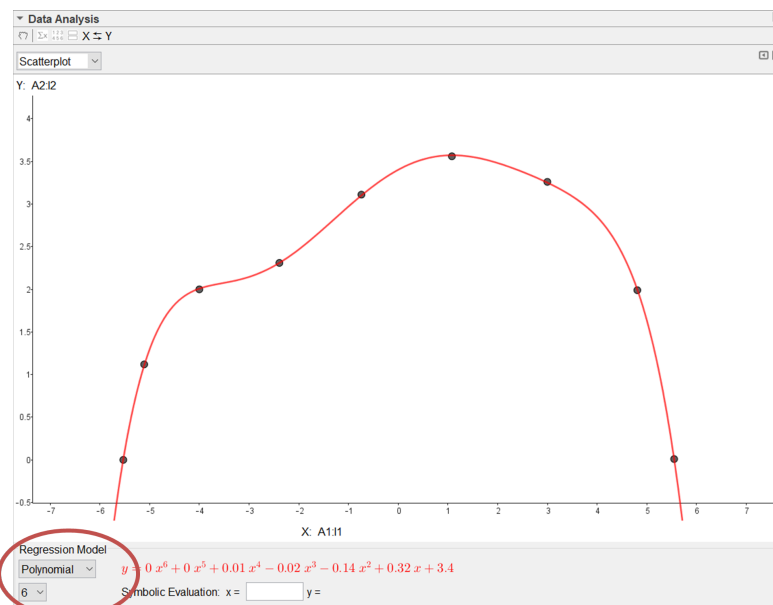| X Value	Y Value
5.53	0.00
-5.11	1.12
-4.00	2.00
-2.39	2.31
-0.74	3.11
1.18	3.56
3.00	3.26
4.81	1.99
5.55	0.01

Figure 8: Image taken from the *Two Variable Regression Analysis* tool. Encircled is where I changed the model to polynomial and the number polynomial it was set at to reach an adequately matching line

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[2]https://wiki.geogebra.org/en/Multiple_Variable_Analysis_Tool

The function I ended up with is:

$$g(x) = (-0.00027x^6 + 0.00021x^5 + 0.00926x^4 - 0.01656x^3 - 0.13954x^2 + 0.31617x + 3.40181)$$

For x in the range:$(-5.5 \leq x \leq 5.5)$

I then used this function to model the avocado in *Geogebra 3D*[3] by using the command "`Surface(g, n, xAxis)`", where *g* is the function, *n*=2π, and *xAxis* tells the program that the surface of rotation is around the X-axis. The completed model of the smaller side of the avocado is shown in Figure 9. Once I had modelled this, I was then able to determine the volume using integration.
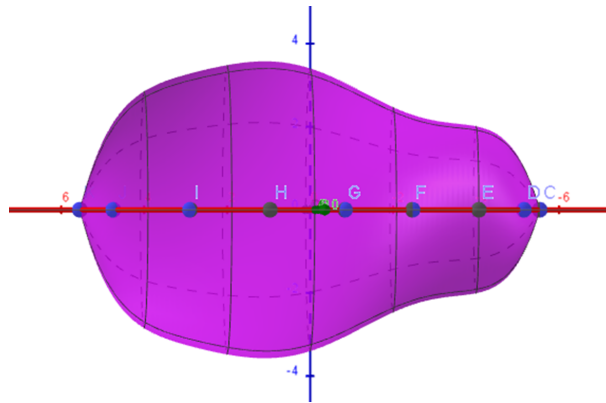

Figure 9: 3D model of the smaller side of the avocado

4   Integration

As we had not begun studying integration yet in class I decided to learn it on my own so that I could more accurately complete my goal. To do this I started by watching some YouTube videos on the basics of integration (ExamSolutions, 2010a) and the rules that govern it.

4.1   Learning How to Integrate

Understanding the basics, I proceeded by attempting a variety of different exercises and examples in order to understand how to integrate fractions, roots and more (Ex-

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[3]https://www.geogebra.org/m/DRfuvhxR

amSolutions, 2010b). I then moved on to videos about definite integrals and curves (Oxford Study Courses, 2021). From there I proceeded to look at examples and do a variety of exercises in order to practice the skills needed to complete my goal.

4.2   Using Integration to Find the Volume of The Avocado

After learning how to integrate, I went in search of a formula that could find the volume of a revolution of the avocado function. If *y* is expressed as a function of *x*, the volume of a solid obtained by a full 360° revolution of the function *y* between the upper and lower limit about the x-axis, is given by the following equation (Math Centre, 2009).

$$a = lower\ limit \quad b = upper\ limit$$

$$\int_a^b \pi y^2\, dx$$

After some further research on the equation, I discovered the law of calculus that states that a constant can be excluded from the integration and re-included at the end (Revision Village, 2020), I therefore rewrote the equation to be the following:

$$V = \pi \int_a^b y^2 dx$$

I found that this simplified my attempt to solve for the volume. I started by inserting the limits and the function into the volume formula.

$$V = \pi \int_{-5.5}^{5.5} ((-0.00027x^6 + 0.00021x^5 + 0.00926x^4$$

$$-0.01656x^3 - 0.13954x^2 + 0.31617x + 3.40181)^2) dx$$

I made several attempts to square the function by hand but struggled to keep track of the small but long numbers I was dealing with. I then decided to use a calculator to correctly square the function. In this step I had to make sure to tell the calculator to round to 10 decimal places, as before I did this many coefficients of x were rounded to

0, resulting in inaccurate values.

$$V = \pi \int_{(-5.5)}^{5.5} ((7.29 \times 10^{-8} \times x^{12}) - (1.134 \times 10^{-7} \times x^{11}) - (4.96 \times 10^{-6} \times x^{10} + 0.00001 \times x^9$$

$$+ 0.00015 \times x^8 - 0.00054 \times x^7 - 0.004 \times x^6 + 0.0119 \times x^5 + 0.072 \times x^4 - 0.2 \times x^3$$

$$- 0.85 \times x^2 + 2.15 \times x + 11.5723) dx$$

Using the rules of integration, the equation becomes:

$$V = \pi \int_{(-5.5)}^{5.5} [\frac{(7.29 \times 10^{-8}) \times x^{13}}{13} - \frac{(1.134 \times 10^{-7}) \times x^{12)}}{12}$$

$$- \frac{(4.96 \times 10^{-6}) \times x^{11)}}{11} + \frac{(0.00001 \times x^{10})}{10} + \frac{(0.00015 \times x^9)}{9}$$

$$- \frac{(0.00054 \times x^8)}{8} - \frac{(0.004 \times x^7)}{7} + \frac{(0.0119 \times x^6)}{6} + \frac{(0.072 \times x^5)}{5}$$

$$- \frac{(0.2 \times x^4)}{4} - \frac{(0.85 \times x^3)}{3} + \frac{(2.15 \times x^2)}{2} + 11.5723x]$$

I then inserted the upper limit (5.5) into the above equation:

$$\pi [\frac{(7.29 \times 10^{-8} \times 5.5^{13})}{13} - \frac{(1.134 \times 10^{-7} \times 5.5^{12})}{12} - \frac{(4.96 \times 10^{-6} \times 5.5^{11})}{11}$$

$$+ \frac{(0.00001 \times 5.5^{10})}{10} + \frac{(0.00015 \times 5.5^9)}{9} - \frac{(0.00054 \times 5.5^8)}{8}$$

$$- \frac{(0.004 \times 5.5^7)}{7} + \frac{(0.0119 \times 5.5^6)}{6} + \frac{(0.072 \times 5.5^5)}{5}$$

$$- \frac{(0.2 \times 5.5^4)}{4} - \frac{(0.85 \times 5.5^3)}{3} + \frac{(2.15 \times 5.5^2)}{2} + (11.5723 \times 5.5)]$$

$$=$$

$$\pi(23.63 - 7.24 - 62.82 + 25.33 + 76.76 - 56.52-$$

$$86.996 + 54.90 + 72.47 - 45.75 - 47.14 + 32.52 + 63.648)$$

$$= 42.792\pi$$

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I then inserted the lower limit (-5.5) into the equation:

$$\pi\big[\frac{(7.29 \times 10^{-8} \times (-5.5)^{13})}{13} - \frac{(1.134 \times 10^{-7} \times (-5.5)^{12})}{12} - \frac{(4.96 \times 10^{-6} \times (-5.5)^{11})}{11}$$
$$+\frac{(0.00001 \times (-5.5)^{10})}{10} + \frac{(0.00015 \times (-5.5)^{9})}{9} - \frac{(0.00054 \times (-5.5)^{8})}{8}$$
$$-\frac{(0.004 \times (-5.5)^{7})}{7} + \frac{(0.0119 \times (-5.5)^{6})}{6} + \frac{(0.072 \times (-5.5)^{5})}{5}$$
$$-\frac{(0.2 \times (-5.5)^{4})}{4} - \frac{(0.85 \times (-5.5)^{3})}{3} + \frac{(2.15 \times (-5.5)^{2})}{2} + (11.5723 \times (-5.5))\big]$$

$$=$$

$$\pi(-23.63 - 7.24 + 62.82 + 25.33 - 76.76 - 56.52 + 89.996+$$

$$54.90 - 72.47 - 45.75 + 47.14 + 32.52 - 63.648)$$

$$= -33.312\pi$$

Following the laws of integration, I subtracted the value found by integrating the lower limit from the value found by integrating the upper limit. Finally, after multiplying by pi I was able to find the volume by hand.

$$V_{total} = \pi(42.792cm^3 - (-33.312cm^3))$$
$$V_{total} = \pi(76.104cm^3) = 239.09cm^3$$

After several attempts by hand, I decided to use a calculator to work out the volume of the revolution as the numbers I was obtaining by hand were either much more or less than what I was expecting. My final hand calculated value ended up being 9% lower than the value calculated by a calculator of $V = 260.0286cm^3$
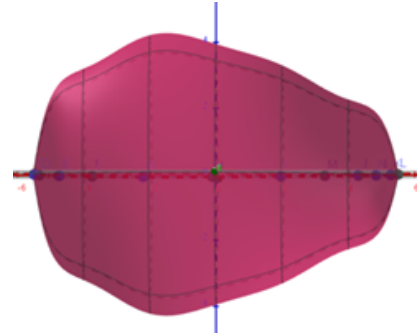
This surprised me at first because it is significantly lower than my previous models' volumes and the volume I measured experimentally. I verified this answer on several other calculators and found the same result. I realised that the reason this result was so small was because I had used the smaller, smoother side of the avocado to model the whole avocado. I then concluded that to come to a proper conclusion I must model the other side and find the volume of an avocado that is entirely the same shape in a revolution as the larger side. By finding this I could then average the two values and

come to a conclusion for the most accurate volume of the avocado.

Using the same method of a two variable regression line, I modelled the larger side of the avocado as seen in Figures 10 (a,b).
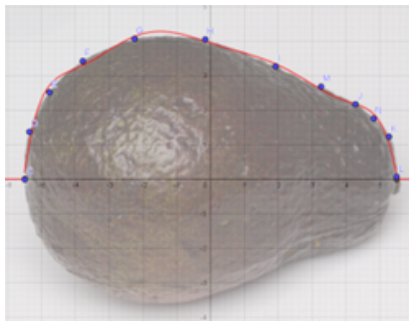
(a) Larger side: regression line   (b) Larger side: 3D model

Figure 10: Larger Side Regression and 3D model

In the larger model the volume was calculated to be 376.2508cm^3. I also verified this using a calculator. This was much closer to what I was expecting based on previous attempts to find the volume, both theoretically and experimentally. I then averaged both determined volume values:

$$V_{Average} = \frac{(260.0286 + 376.2508)}{2}$$
$$V_{Average} = 318.14cm^3$$

This value is 10% lower than the experimental value in Section 2, but deemed acceptable, as the experimental measurement was not particularly accurate.

5   Finding the Volume using an Alternative Equation

During my research I came across a paper that proposes an equation to find the volume of a *Hass* avocado (Erickson & Kikuta, 1965). The equation seems to be based around the combination of several shapes that can be found within the avocado, such as a circle/sphere (outlined in red on Figure 11) with a formula of:

$$\frac{4}{3}\pi r^3 = volume\ of\ a\ sphere$$

The full equation for the volume of an avocado is as follows:

$$\pi[\frac{(d_1^3 + d_2^3)}{12} + r^2h_1 + \frac{h_2}{3}(r_1^2 + r_1r_2 + r_2^2)]$$

Where, as seen in Figure 11:

d_1 Diameter at r_1 from the large end

d_2 Diameter at r_2 from the small end

d_m Mean maximum diameter

h_1 Distance between $d_{1Bottom}$ and d_{1Top}

h_2 Distance between d_{1Top} and d_2

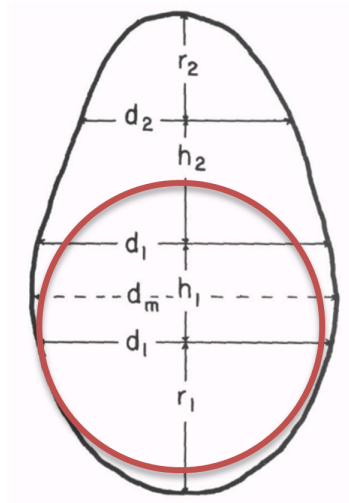Radii $r_1 = \frac{d_1}{2}$ $r_2 = \frac{d_2}{2}$ $r = \frac{(d_1 + d_m)}{4}$

Figure 11: Alternate Model after (Erickson & Kikuta, 1965)

I taped twice around the long axis of the avocado, separated by 90°, and using digital calipers, measured the diameter every centimetre for both lines down the entire length of the avocado. I averaged these values, with the results given in Table 3.

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Table 3: Measurements for Eriksen & Kikua Model

D	d_1=7.5cm	d_2=4.13cm	d_m=7.6cm
H	h_1=1.5cm	h_2=4.75cm	
R	r_1=3.75cm	r_2=2.065cm	r=3.78cm

Substituting values from Table 3 into my equation, I found a final volume.

$$\pi[(7.5cm)^3 + \frac{(4.13cm)^3}{12} + (3.78cm)^2 \times 1.5cm + \frac{4.75cm}{3}$$
$$\times((3.75cm)^2 + 3.75cm \times 2.065cm + (2.065cm)^2)]$$

$$=$$

$$\pi[40.27cm^3 + 21.4cm^3 + 1.58cm(26.064cm^2)]$$

$$=$$

$$\pi(102.85cm^3) = 323.11cm^3$$

6 Evaluation

In the previous sections I have used several methods to find the volume of my sample avocado. I have not yet discussed the benefits and limitations of using these methods. Measuring the volume experimentally, especially with such a relatively large object must be done carefully. Since I did not have particularly accurate measurement tools, I had to try to make the best estimation possible of the change in measurement when I pushed the avocado underwater in a measuring jug. The benefit of doing this is that it gave me a rough idea of what the volume will be but I have to use the measurement cautiously, meaning that I would not trust it to be my final value. Using existing shapes to find the volume is beneficial because it avoids the issue of trying to build your own functions. This does however prevent one from building an accurate model to work from. On the other hand, it has been done effectively in the paper mentioned above about finding the volume of an avocado (Erickson & Kikuta, 1965), they have taken a variety of shapes and their formulae, including a hemisphere, cylinder and the frustum of a cone, to build an equation that fits many measurements of one avocado. My hand

calculated volume of the smaller side of the avocado has a 9% percentage difference to the calculator calculated result. I think this is acceptable as a calculator can work more effectively with numbers that have many more decimal places than I can handle by hand. I think that if I had continued to solve the equation, starting with more decimal places in the polynomial coefficients I would have achieved a more accurate answer, with a smaller percentage difference to the calculator. Finally, creating a model of each side of the avocado using a regression line has its drawbacks as well. As can be seen in Figure 12, the regression line did not necessarily line up with the edge of the avocado which could have led to an error in the calculations.
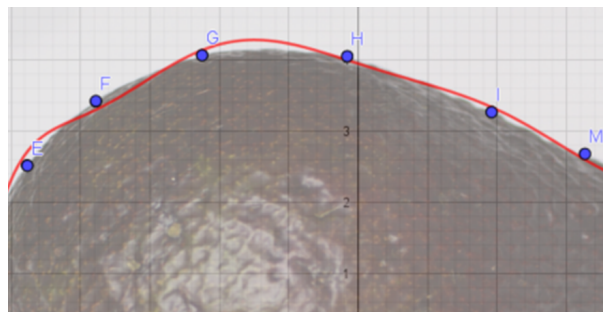

Figure 12: Misaligned regression line on the larger side model of the avocado

Using a regression line however was extremely useful in finding the most accurate volume possible. Taking the average of both sides gave me a much better value than I would have obtained if I had only used one side's value. In a further investigation I would also find the volume of the avocado pit and subtract that from my final volume to get a better understanding of how much avocado fruit I can use when making toast in the mornings. I would also experiment with several other avocados to look at the range of sizes of avocado fruit versus the pit and find the optimum size for an avocado. In further investigation I would also look into creating my own equation to find the volume with any measurements or to find a function that could model the avocado simply by changing several values.

The value I obtained by averaging the results across the two integrations, and the value obtained using the alternative equation were very close. Due to this I would conclude that the volume of this avocado is approximately between 318.14cm^3 and 323.11cm^3.

This seems to be an acceptable approximation in comparison with the uncertain experimental test. Throughout this exploration I have enjoyed learning about an area of mathematics on my own and then testing it with something that I am interested in. I hope to in the future continue to build my knowledge of mathematics with and without a teacher and apply it to my other interests and my future career in engineering.

References

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