Flight Path Analysis: Modeling the Dynamics of Badminton Shots with Python

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Subject: Naturvitenskap

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I - ABSTRACT AND INTRODUCTION

Abstract

This report investigates the impact of launch speed and angle on the trajectory of a badminton shuttlecock during a clear shot and has created new and original code in order to model it effectively. The study integrates theoretical analysis and experimental data collection to understand the physics of projectile motion in badminton. The theoretical framework involves solving differential equations governing shuttlecock motion under various drag force models. Analytical solutions are explored for linear drag (n=1) and numerical solutions are employed for quadratic drag (n=2). Comparison between theoretical predictions and experimental outcomes reveals insights into the accuracy of the models and the influence of environmental factors. The analysis suggests optimal launch conditions for achieving maximum shuttlecock range and effectiveness in a clear shot. The findings also contribute to the broader understanding of physics in badminton and provide insights for optimizing gameplay strategies and equipment design, and suggest topics for further research.

Introduction

As an aspiring badminton player and scientist, I've always been fascinated by the physics of projectile motion. With my extensive experience in the sport and my background in International Baccalaureate Physics, I will investigate the following research question: *how does launch speed and angle impact the trajectory of a badminton shuttlecock during a clear shot?* In badminton, the clear shot is one that hits the shuttle high and long. In this essay, I will start with the governing differential equations for badminton motion and solve them analytically and numerically. I will conduct shuttlecock experiments to record trajectories, and compare with my theoretical analysis.

A shuttlecock is a projectile that experiences both horizontal and vertical acceleration due to the forces acting on it. During a clear, the shuttlecock creates a shape similar to a quadrilateral. The force of air resistance acts in the opposite direction of motion (and is depicted as splitting into two different forces). This can be observed in fig-1.

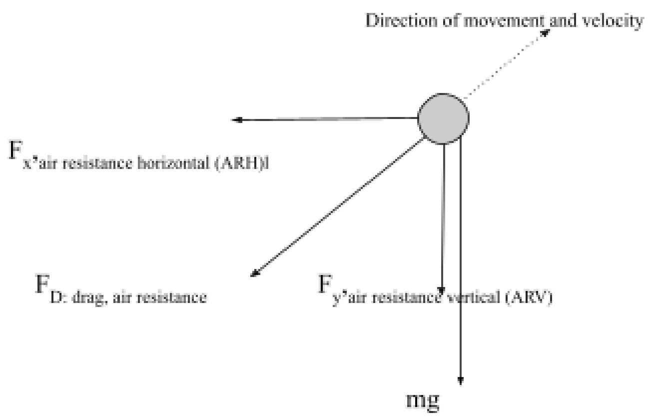


Figure 1: Dissociation of forces experienced by badminton ball during a clear shot

As the shuttle travels upward, air resistance slows it down, causing it to fall at a steeper angle than it would under ideal conditions (i.e., gravity only). For the best trajectory, players typically aim for a high and long clear, which is crucial in determining its effectiveness. An image of a clear shuttle trajectory is shown in fig-2.

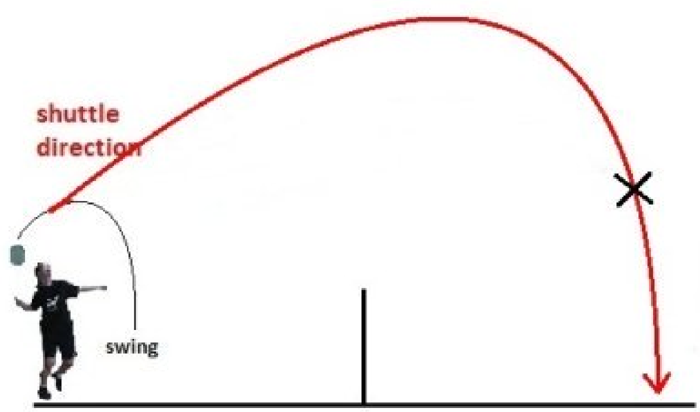


Figure 2: A typical trajectory of a clear for a badminton ball (Master Badminton, 2016)

The force applied to the shuttlecock by a racket determines its initial velocity and direction. The shuttlecock's trajectory, governed by Newton's second law, is affected by air resistance, which can be proportional to either its velocity or velocity squared. The complexity of my project arises from the velocity-dependent air resistance force. If air resistance is insignificant, the trajectory can be described by a simple quadratic function. The governing equation with the linear dependence of air resistance force is also analytically solvable as shown later in my thesis. When the air resistance force is proportional to velocity squared, there is no analytical solution, and a numerical solution using the Euler method is employed.

For the experimental work of trajectory tracking, the difficulties lie in producing and measuring the initial high launching speed and angle. Different strategies have been tried out, and the measured trajectories are compared with my theoretical calculation results.

II - THEORY

When the shuttlecock is in flight Newton's Second Law can be reformulated as so:

$$\vec{W} + \vec{F}_D + \vec{B} = m\vec{a} \qquad (1)$$

The sum of forces is equal to the sum of the gravitational force, drag force and buoyancy. In this equation, B (buoyancy) can be neglected as it has a negligible impact on the trajectory. If a shuttlecock is falling down vertically without any horizontal movement, the speed and resistance force will increase until the sum of forces is zero in the vertical-direction. This is the point the shuttlecock reaches terminal velocity, and moves with zero acceleration. At terminal velocity, weight will be equal and opposite to the drag force. According to Thornton and Marion (2003), the magnitude of air resistance force can be expressed as:

$$F_v = bv^n \qquad (2)$$

where n is a real number and b is a constant that depends on the properties of the air. The direction of air resistance force is opposite to the direction of the shuttlecock velocity. n is related to the velocity of the object in that fluid. b can be defined as:

$$b = \frac{1}{2}\rho S C_D \qquad (3)$$

ρ denotes air density, which varies with temperature and pressure. C_D, the drag coefficient, represents the aerodynamic characteristics of the object's shape, such as the shape of the object and surface roughness. S is the surface area of the object that is exposed to the air drag.

A differential equation relates one or more unknown functions and their derivatives. Equation (1) is a differential equation governing the motion of a shuttle. This is the equation I will be predominantly discussing in this essay. This can be done by looking at the dissociations of forces in horizontal and vertical directions for a badminton shuttlecock, which was shown in figure. 1. This may be written in equation format as such:

$$F_x = ma_h = F_D sin\Theta \tag{4}$$

$$F_y = ma_v = F_D cos\Theta - mg \tag{5}$$

Both directions will have their own accelerations, which we can use to find the velocity and then the displacement. For case n=2, this will yield a differential equation that can only be solved with a numerical integration method, Euler's method. When n=1 the equation can be solved analytically.

CASE 1: n is equal to 1.

When n is equal to 1, $F_D \propto v$. This leads to linear drag, which applies to positional movement. A higher value of drag will cause the shuttle to come to rest more quickly. The different forces within F_d, the linear drag force, can be split into $F_{dx} = - bv_x$ and $F_{dy} = - bv_y$, as velocity is also a vector and hence has vector properties (fig-3).

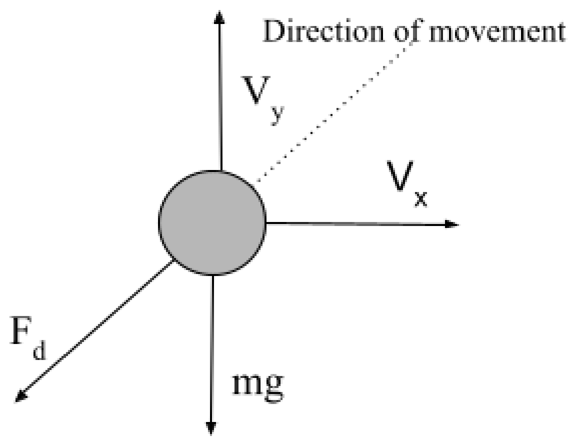


Figure 3: The dissociation of velocity of a badminton shuttle in air.

It is possible to to write Equations (4) and (5) as below when n=1:

$$m\,a_x =-\ bv_x \tag{6}$$

and

$$m\,a_y = \ -\ mg\ -\ bv_y \tag{7}$$

Equations (6) and (7) can be solved analytically. The method to solve is seen in Appendix. A. The solution is seen below.

$$x(t)\ =\ x_0\ +\ v_0\frac{m}{b}(1-e^{\frac{-tb}{m}}) \tag{8}$$

$$y(t)\ =\ y_0\ +\ v_T\ +\ (e^{\frac{-tb}{m}}\text{-}1)(v_T\frac{m}{b}\text{-}\frac{V_0 m}{b}) \tag{9}$$

The equations suggest that the shuttlecock's horizontal trajectory is affected by both the initial launch conditions (x_0, y_0, v_0, Θ) and the decelerating effect of air resistance. It predicts a trajectory that starts with the initial velocity but exponentially slows down due to air resistance. These two equations allow us to calculate the trajectory of the shuttlecock by plotting down the points that the common variable *t* creates.

CASE 2: n is equal to 2

When n = 2, the drag force becomes:

$$F_D = \frac{\rho S C_D v^2}{2} \tag{10}$$

This is also seen by Cooke & Firoz, who measured the aerodynamics of several shuttlecocks and studied them in a wind tunnel.

By bringing Eq.(10) into Eq.(1), the following differential equation is obtained:

$$m\frac{d\vec{v}}{dt} = m\vec{g} - \frac{1}{2}\rho S C_D \vec{v}^2 \tag{11}$$

This is the equation studied by Cohen. I developed a Python Code on Spyder, which gives a numerical solution to Eq.11 that one may compare to the experimental value. The numerical solution is obtained by using Euler's method to integrate Eq.11 twice for a very small time step *h* repeatedly. The first integration is from acceleration to velocity, and the second from velocity to displacement (trajectory), and then appending all the results into a list.This makes it possible to plot out sufficient points to join together to make a trajectory.

The Euler method is used as there is no analytical solution to the equation. The reason there is no analytical solution is because the horizontal and vertical velocities now influence each other. When n = 1, v_y does not appear in the equation describing v_x. When n = 2, it is impossible to separate the variables from each other such that *t* is on one side and everything else is on the other like demonstrated in Appendix. A for the case of n = 1.

According to previous equations (4) and (5), the horizontal acceleration component of the shuttle follows the equation below:

$$a_x = \frac{-F_D \times cos(\frac{\theta}{180 \times \pi})}{m} \tag{12}$$

and the vertical acceleration follows the equation below:

$$a_y = \frac{-F_D \times sin(\frac{\theta}{180 \times \pi})}{m} - g \tag{13}$$

where drag is equal to

$$0.5 \rho S C v^2 \tag{14}$$

(according to Cooke & Firoz's calculations). This is used for the situation in which the shuttlecock is launched with a starting speed and angle.

For the Python code, velocity and drag force are both updated because it keeps changing due to the movement of the shuttlecock. The figures are then plotted against time to show each individual movement. It may also be observed that there is a vertical scale correction factor to negate the effect of the filming angle (fig-4).

```python
import matplotlib.pyplot as plt
import numpy as np
import csv

#variables
g = 9.81 #ms2
p = 1.188 #air density kgm3
D = 0.06  #Surface Area m2.
S = 3.14*(D/2)**2
C = 0.65 #drag coefficient Newtons
m = 0.00475  #mass kg
V =  44.257#inital velocity m/s
ang = 60 #inital angle, in degrees
vertical_scale_factor = 1.45
exponent = 2 #exponent used in calc.

drag=0.5*p*S*C*V**exponent
t = [0]                         # list to keep track of time
vx = [V*np.cos(ang/180*np.pi)]  # list for velocity x and y components
vy = [V*np.sin(ang/180*np.pi)]
x= [0]
y = [0]
h=0.01 # second, time step
ax = [-(drag*np.cos(ang/180*np.pi))/m]
ay = [-g-(drag*np.sin(ang/180*np.pi)/m)]
# uses the euler method twice for both to get s.x and s.y
n=0
while (y[n] >= -1.75):      #or height at landing!!
             # Check that the last value of y is >= 0
    t.append(t[n]+h)

    #velocity
    vx.append(vx[n]+h*ax[n])  # Update the velocity
    vy.append(vy[n]+h*ay[n])

    #distance
    x.append(x[n]+h*vx[n])
    y.append(y[n]+h*vy[n])
    #SO VELOCITY IS
    vel = np.sqrt(vx[n+1]**2 + vy[n+1]**2)

    drag=0.5*p*S*C*vel**exponent
    ax.append(-(drag*np.sign(vx[n])*np.cos(ang/180*np.pi))/m)
    ay.append(-g-(drag*np.sign(vy[n])*np.sin(ang/180*np.pi)/m))
    n=n+1
    #ang = np.arctan(vy[n]/vx[n])

plt.grid(1)
plt.plot(x,y,'co', label="Predicted Model",markersize=1)
plt.legend(loc="upper right")
plt.ylabel("Height (m)")
plt.xlabel("Distance Travelled(m)")
plt.title('Pathway Travelled', fontsize=16)
plt.axis('on')
```

Figure 4: Python code calculating the trajectory of a badminton shuttle using Euler's method.

This model implementation predicts how the shuttlecock may travel. An example is below, using a drag of 0.65 (as suggested by Cohen), a starting velocity of 44.3 m/s and a launch angle of 60°. This yields the following prediction (fig-5).

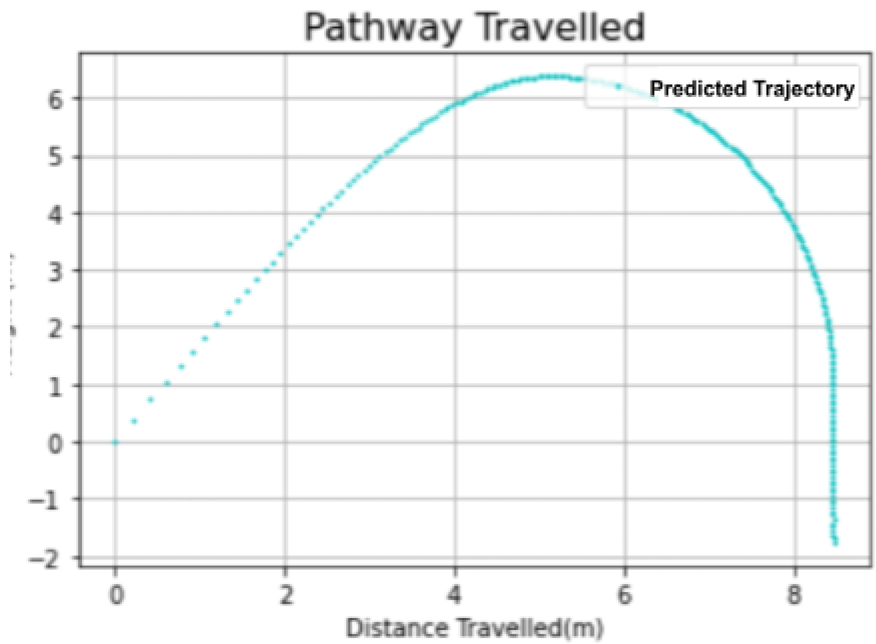


Figure 5: the predicted trajectory of badminton ball at V_0 = 44.3 m/s and Θ_0=60

III- EXPERIMENT

There were multiple sources from which experimental data was collected, due to the development of research and the gradual accumulations of my experiences. Data was collected by filming the trajectory. Two post-it notes marked a meter's distance, so that I could put the video into a data processing software called Pasco Capstone software (Pasco) and adjust it for scale correction due to filming project angle. This allowed me to use Pasco to find the ranges of distance and plot and then visualize the trajectory.

The first set of experimental data was from using an improvised pipe that I could build pressure in (fig-6). It had a valve that could be pulled to release the air pressure built up by a pump as a force and the pipe's angle can be adjusted. The precise speed was found later with Pasco while the launching angle can be prescribed and measured. In this experiment, a bicycle pump was used which was equipped with a pressure gauge with an uncertainty of ± 5 PSI.

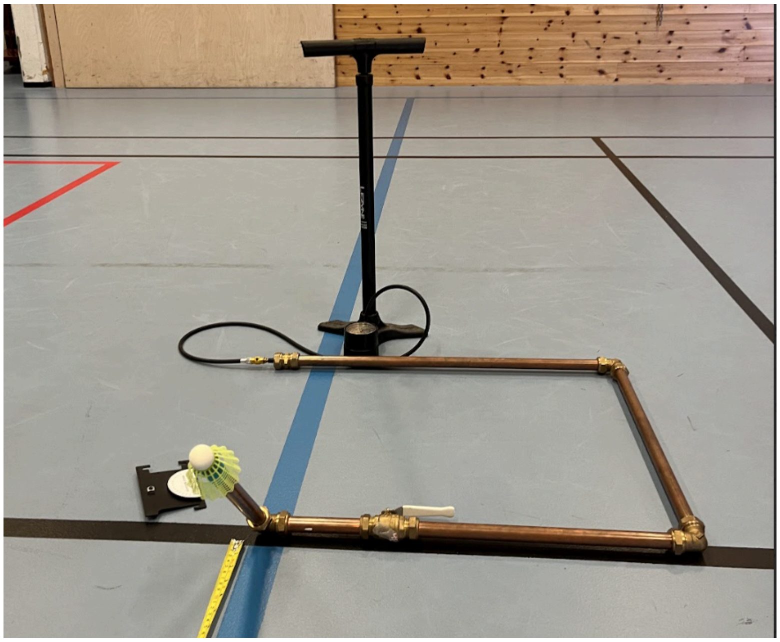


Figure 6: Image of badminton ball launcher, the bicycle pump, and the method of measuring the angle

To test how the launch angle impacts the trajectory, I had to build the pieces that would be part of the launched badminton shuttlecock. The goal of this construction is to create an assisting piece for the launch of the badminton shuttlecock, so that it will fit over both the brim of the pipe (circumference 7.3cm ± 0.05cm) and fit within the badminton shuttlecock (top circumference 6.9cm) (fig-7).

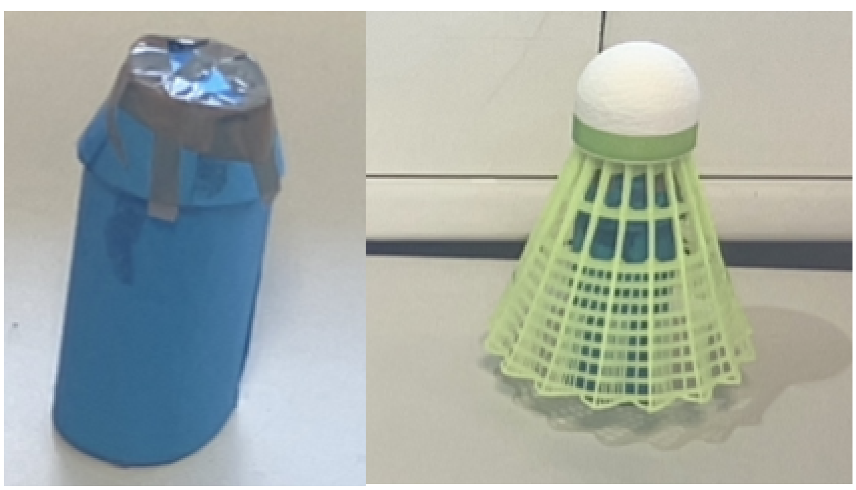


Fig 7: *My badminton shuttles with the supporting piece (blue)*

In this experiment, blue paper was used to illuminate the shuttlecock more clearly when in flight. Preliminary testing revealed that white shuttlecocks did not show up well against white walls.

I encountered the opportunity to have my badminton shuttlecocks shoot out from an automatic feeder, the Knight Trainer Pro. The previous equipment was only able to launch the ball at low speeds, so the automatic badminton feeder was able to expand the range of variables for this experiment (up to 44.3m/s) (fig-8). This is my second source of experimental data.

Figure 8: the badminton automatic feeder and brand of badminton ball

If the trajectory should be measured according to the first badminton ball launcher, it would need to be changed as the height of launch depends on the angle. The following code adjusts the launch height by the launch angle with trigonometry, as the length of the pipe is measured and the angle of launch is known (fig 9).

```
starting_point = - np.cos(ang/180*np.pi)*0.40
```

Figure 9: Code that adjusts the height depending on the angle of launch.

These two experimental methods provide a basis for how the trajectory of a badminton ball may change due to a change in launch speed or launch angle. The collected data will be used to compare with my python code calculation results.

Other than equipment, it was also crucial that I note the range of launch speed. When picking values for the launch speed of the shuttle, I had taken into account Cooke & Firoz's calculations, where they stated that the drag coefficient C_D is approximately constant for Reynolds numbers between 1.0×10^4 and 2.0×10^5. Reynold's number is a scalar quantity that helps predict flow patterns in fluids by quantifying the ratio between inertial and viscous forces. At low Reynolds numbers, flow tends to be laminar. At high, turbulent. Reynold's number shall be considered for calculating trajectories, as it impacts the value of the drag coefficient (Wikipedia Contributors, 2023).

Using the equation $Re = \frac{DV}{v}$, (where D is air density, V is speed and v is air kinematic viscosity). D is 2.5cm. Air kinematic viscosity was calculated with the same information to be $15.43 \times 10^{-6}\, m^2/s$), we can calculate that the smallest velocity for this would be $\frac{1.0\times10^4\times15.43\times10^{-6}}{2.5\times10^{-2}}$=6.172 m/s. The highest would be 123.44 m/s. The values I picked were within this range, and this topic will be further discussed below.

The same badminton shuttle brand has been used for all launches.

IIII - RESULTS:

The following table shows the average measured values of the experiment (starting velocity for those that have it, average range for both), the errors and standard deviation.

Table A. 1: How Launch Angle impacts distance launched (PSI 60 ATP) using the pump launcher, table part 1

Launch Angle	Launch speed (m/s) ± 0.02			Average	Standard Deviation
	R.1	R.2	R.3		
10.0	9.83	8.87	9.37	9.36	0.5
20.0	10.54	10.11	10.67	10.44	0.3
30.0	9.27	8.42	8.24	8.64	0.6

40.0	8.71	9.03	9.93	9.22	0.7
50.0	11.24	8.62	9.42	9.76	1.3
60.0	10.76	9.74	11.12	10.54	0.7
70.0	8.87	9.36	11.49	9.91	1.3

Table A. 2: How Launch Angle impacts distance launched (PSI 60 ATP) using the pump launcher, table part 2.

Launch Angle	Launch Distance (m) ± 0.01			Average	Standard Deviation
	R1	R2	R63		
10.0	3.32	3.02	3.35	3.23	0.4
20.0	4.52	4.43	4.34	4.43	0.1
30.0	4.03	4.55	4.14	4.24	0.3
40.0	4.94	4.11	4.90	4.65	0.5
50.0	5.91	4.62	5.16	5.23	0.7
60.0	5.56	4.01	3.75	4.44	1.0
70.0	3.69	5.12	5.38	4.73	1.0
80.0	5.38	5.44	3.94	4.92	0.7

As expected, it can be seen in table A1 that the launch speed is approximately constant for all launch angles, as the pressure was constant. It can also be seen that launch distance is the highest for 50°.

Table B.1: Variation of launching PSI (speed) with launch distance

PSI	Speed (m/s) ± 0.02			Average (m/s)	Stan. Dev.	Distance Traveled (m)± 0.01			Average (m)	Stan. Dev
	Repeat 1	Repeat 2	Repeat 3			Repeat 1	Repeat 2	Repeat 3		
20	4.93	5.81	5.19	5.31	0.5	2.17	2.72	2.79	2.56	0.3

30	6.76	7.12	6.88	6.92	0.2	3.28	3.131	3.759	3.39	0.2
40	7.99	9.53	8.97	8.83	0.8	5.03	4.25	5.36	4.88	0.5
50	12.36	11.56	11.6	11.84	0.4	5.24	4.26	4.63	4.71	0.5
60	10.71	9.77	11.17	10.55	0.7	5.51	4.01	3.74	4.42	1.0
70	10.92	10.39	10.58	10.63	0.3	3.67	5.17	5.35	4.73	1.0
80	11.93	10.80	12.4	11.71	0.8	5.38	5.48	3.96	4.94	0.7

Changing speed, on the other hand, can be observed to change the launch speed. It also impacts the distance traveled.

Table C. 1: The variation of horizontal displacement with angle using the badminton ball feeder (with the launch speed 44.3 m/s).

Angle	Horizontal displacement from badminton launcher (m) ± 0.01											Average	Stan. Dev.
45	14.58	15.18	15.4	15.62	15.63	16.18	16.29	16.44	16.59	16.83	16.97	17.06	16.06	0.8
30	15.97	16.42	16.97	17.26	17.61	18.14	18.35	18.54	-	-	-	-	17.41	1.1
20	16.73	17.01	16.98	17.14	17.36	17.63	-	-	-	-	-	-	17.14	0.3
15	16.64	16.98	17.33	17.55	17.67	17.73	17.82	-	-	-	-	-	17.39	0.4

Using the other badminton launcher, it is evident that the smaller the angle, the further the ball goes.

When the Standard Deviation ⩾1, there may be some variation, while a Standard Deviation < 1 can be considered low (Winters. R, 2012). Preliminary analysis therefore shows four anomalies, but taking into account the quality of equipment used, I will not discard any of the data.

In addition, the data I first collected using the pipe usually never reached a high enough velocity to mimic an actual clear. I made the adjustment of using a different method, which may be limiting as there are no environmental controls.

Error:

Calculations can be done to calculate the error of speed as the error of the distance is found with the ruler. According to *Physics For the IB Diploma*, when $Q = a \times b$, $\frac{\Delta Q}{Q} = \frac{\Delta a}{a} + \frac{\Delta b}{b}$.

Although the values for speed are not derived by hand, the error can still be found using the percentages. Using average values for speed and for distance, it can be found that:

$\frac{0.01}{4.48375} = 0.00223 = 0.223\%$

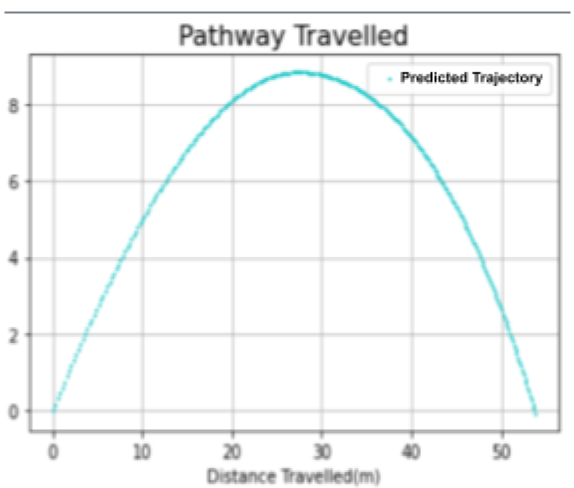
This means that the decimal error for speed = $0.2\% \times 9.696 = 0.0216 = \pm 0.02$

The same calculation can be done for Table-B.1

V- ANALYSIS

In this section, I will first compare my experimentally measured trajectory, my theoretical calculations from Python code and the reported results from other researchers. I will also look more specifically at the details that may also describe the trajectory, such as the range of the badminton shuttle. The following figures are produced with Pasco by me and are some examples to illustrate the result of collecting the data points from the recorded videos. In Pasco, it is possible to display the trajectory for different angles and different velocities and therefore observe the impact of launch speed and angle.

Previously, it was discussed that $F \propto v$ or $F \propto v^2$. The best calculation results can be seen using the comparisons between these two.



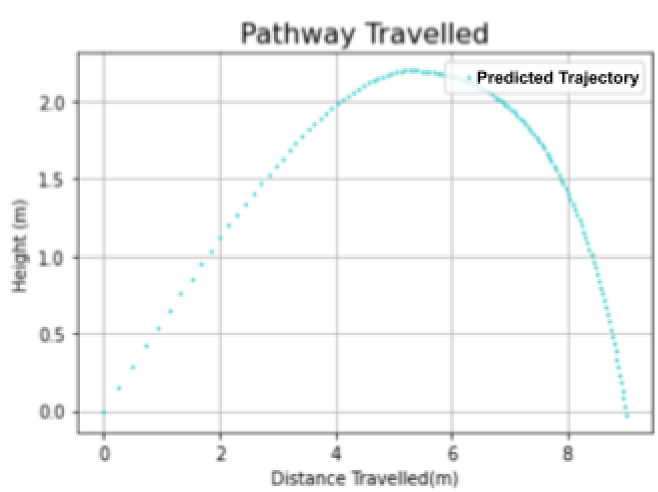


Fig 10: Trajectory calculated under the conditions θ_i = 30°, v_i = 30 m/s for F= bv (left) or bv^2 (right)

From my Python code calculations, it can be observed that there is an extreme difference in range. As previously described, the length of a badminton court is 13.42 meters. In other words, at θ_i = 30° and assuming the flying shuttlecock lands inside the court, it can be judged intuitively that the relationship between air drag resistance force and the speed of a shuttlecock is not linearly proportional to the speed of a shuttlecock (i.e. F= bv). This can be verified by examining the trajectory measured by Chen et al.

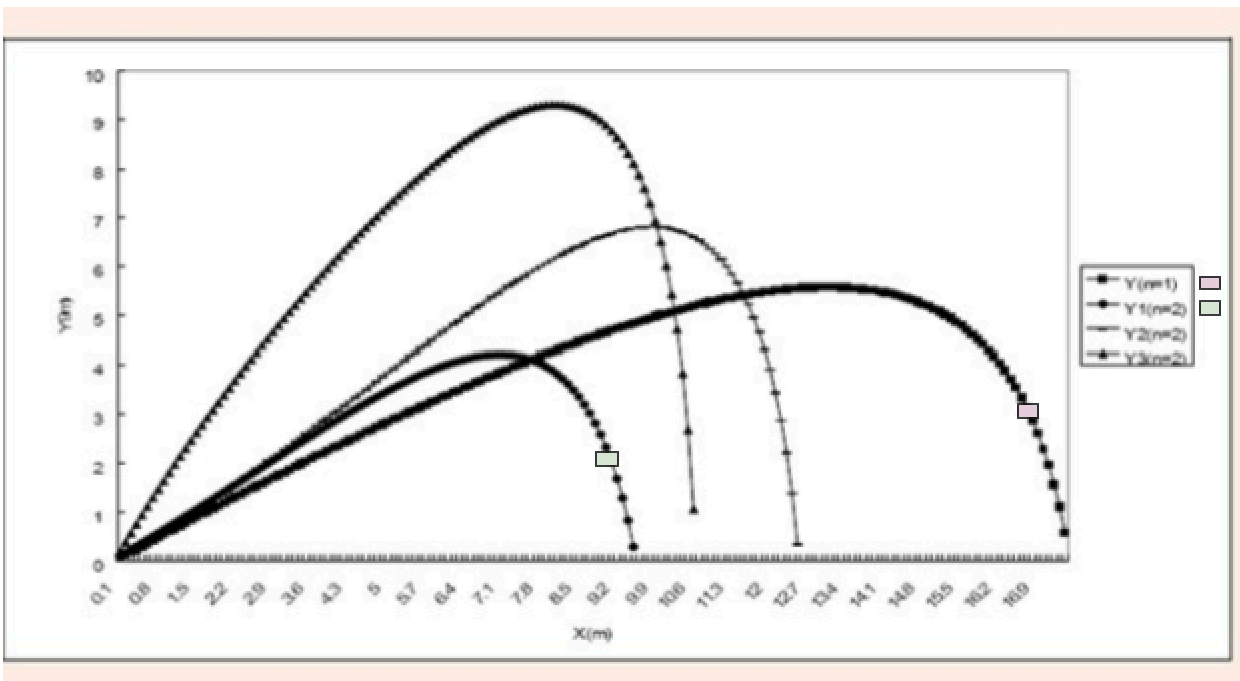


Fig 11: Cohen's graph for the trajectory of the shuttlecock at different angles with different powers- Relevant lines are marked with pink (bv) and green (bv^2).

In the above graph, the box on the right describes four variables. Here, it is only necessary to look at the first two, written as Y(n=1) (marked green) and Y1(n=2) (marked pink). In the figure below both have same initial velocity v_i = 30 m/s, θ_i = 30° but are calculated according to

$F = bv^2$ and $F = bv$ respectively. The trajectories of the shuttlecock are above, and it is evident that the trajectory of bv is longer.

Experimentally, I did not perform v_i = 30 m/s, θ= 30°. I did do v = 44 m/s, θ= 30° (fig-12). As one can observe, it does not go out, with a total distance of 8.5 meters. It is therefore realistic to assume that launching at 30 m/s can only yield a shorter distance of launch.

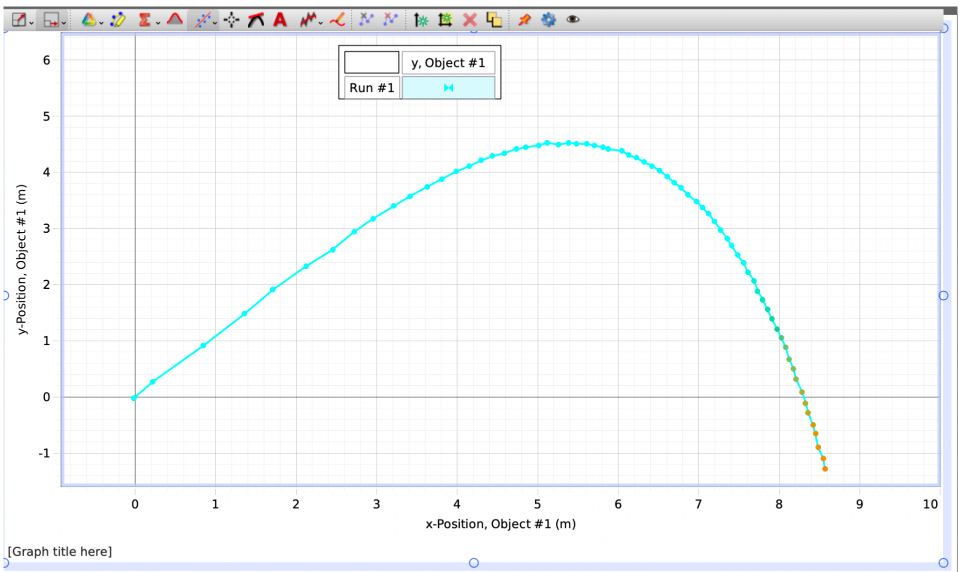


Fig 12: Measured trajectory shown at v= 44 m/s, θ= 30° on Pasco

Therefore, the calculation using the linear drag force relation predicts much longer ranges for the shuttle. The launch speed is the determining factor for the instantaneous speed at all points once this relationship is established. With this in mind, it is possible to observe and compare experimental and theoretical trajectory ($\propto v^2$).

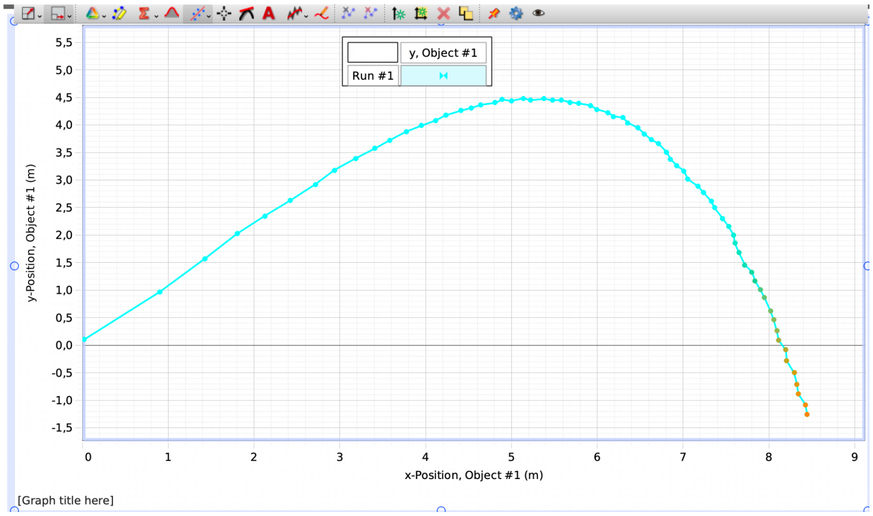


Figure 13: *Measured trajectory at v= 44 m/s, θ= 20°*

For example, Fig. 13 is the trajectory at the same speed but with a different angle of 20°. It is possible to observe that the overall shape is similar but that the one with the lower angle traveled a shorter distance. After mapping out the average distance traveled by with different angles, the following result may be seen.

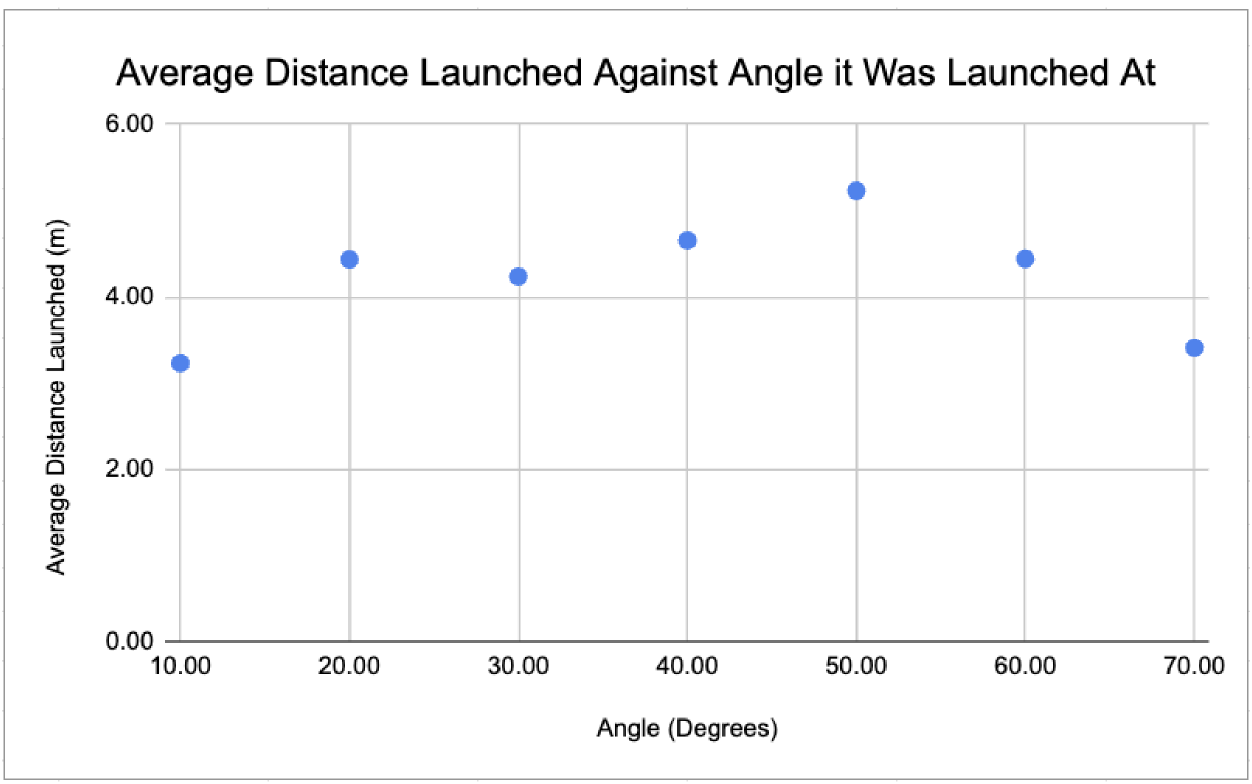


Fig 14: Measured average distance behind the net a badminton clear shuttle landed at 44 m/s

As can be observed from Fig. 14, the highest shuttle travel distance occurs around 40-60°. In the absence of air resistance, a 45-degree launch angle maximizes the range (horizontal distance traveled) of a projectile for a given initial speed. This is because at 45°, the vertical and horizontal components of the initial velocity are equal, and this allows the projectile to stay in the air for the longest time. However, when we take into account air resistance, the angle for maximum range < 45°. This is because at lower launch angles, the vertical component of velocity is smaller, and the projectile spends less time in the air, which means it is exposed to air resistance for a shorter duration.

Comparing the first graph (fig-12) to the following graph, the impact of change in speed can be observed.

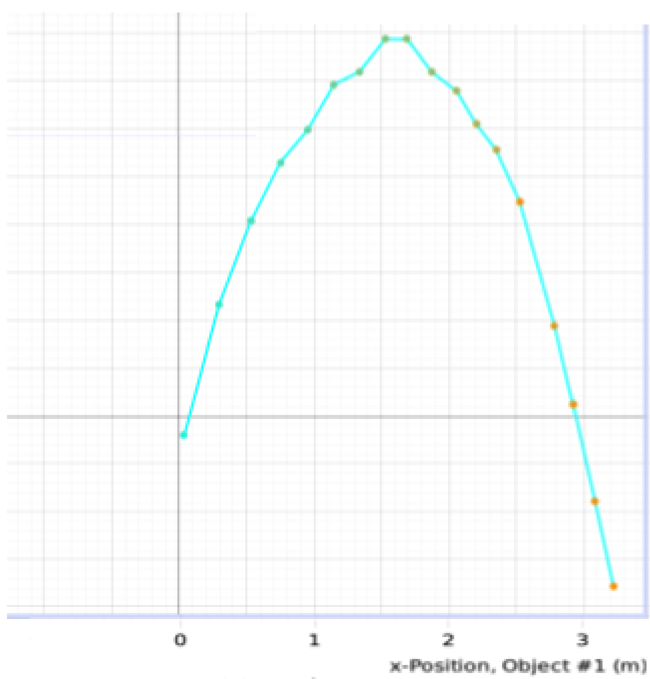


Fig 15: Measured trajectory shown at v= 11 m/s, θ= 20° on Pasco

It is evident that distance increases the higher the initial velocity is.

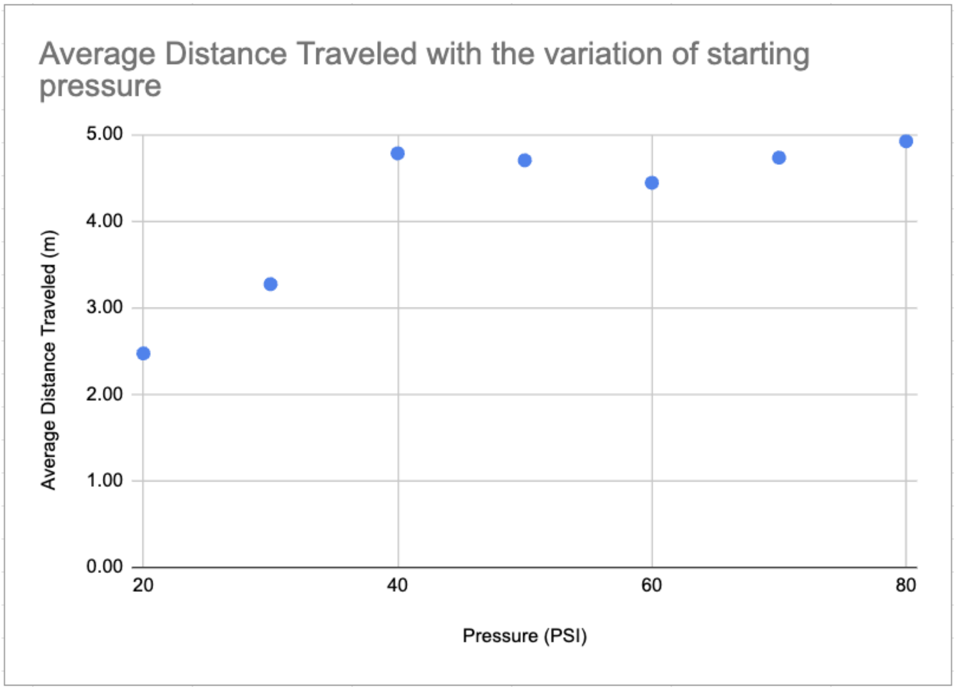


Fig 16: Average distance measured behind the net a badminton clear shuttle landed with angle 30°

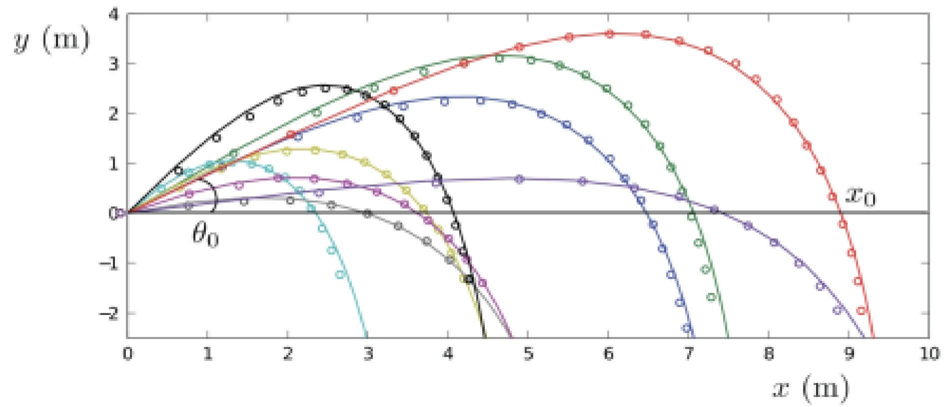
However, something interesting to note about the shape of Fig-15 is that this one is more quadrilateral-shaped. The higher the launch speed, the clearer the aerodynamic wall, which is usually shown as an asymmetrical trajectory ending with an almost vertical fall. Looking at Fig-10 it may also be seen that the aerodynamic wall is only present when $F \propto v^2$. This will be discussed in Evaluation. Overall, it is possible to observe the impact of launch speed and angle on the trajectory separately. By considering both of these scenarios simultaneously, it is possible to model and predict the influence of these two factors on any trajectory of a badminton shuttle.

VI- EVALUATION

VI.1- Comparison With Reported Trajectories and the Tuning of the Drag Coefficient

Despite the attempt to take Reynold's number into account in my theoretical analysis, this was not research that was conducted specifically in this essay. In addition, the value of Reynold's number is uncertain in my experiments. Therefore, only qualitative discussions on Reynold number have been performed..

Second, it can be noted that there are some discrepancies within the calculation results. Below, I have displayed three trajectories to compare to Cohen's.



. Comparison between the observed trajectories (circles) and trajectories calculated with a pure drag equation and $\mathcal{L} = 4.6$ m (solid line) for different initial conditions: $U_0 = 19.8$ m s^{-1} and $\theta_0 = 39°$ (blue); $U_0 = 24.7$ m s^{-1}, $\theta_0 = 44°$ (green); $U_0 = 6.8$ m s^{-1}, $\theta_0 = 55°$ (cyan); $U_0 = 9.7$ m s^{-1}, $\theta_0 = 44°$ (yellow); $U_0 = 9.5$ m s^{-1}, $\theta_0 = 30°$ (light-purple); $U_0 = 9.6$ m s^{-1}, $\theta_0 = 18°$ (gray); $U_0 = 13.4$ m s^{-1}, $\theta_0 = 58°$ (black); $U_0 = 37.6$ m s^{-1}, $\theta_0 = 38°$ (red); $U_0 = 32.3$ m s^{-1}, $\theta_0 = 12°$ (dark-purple).

Fig 17: Cohen's comparison between observed trajectories.

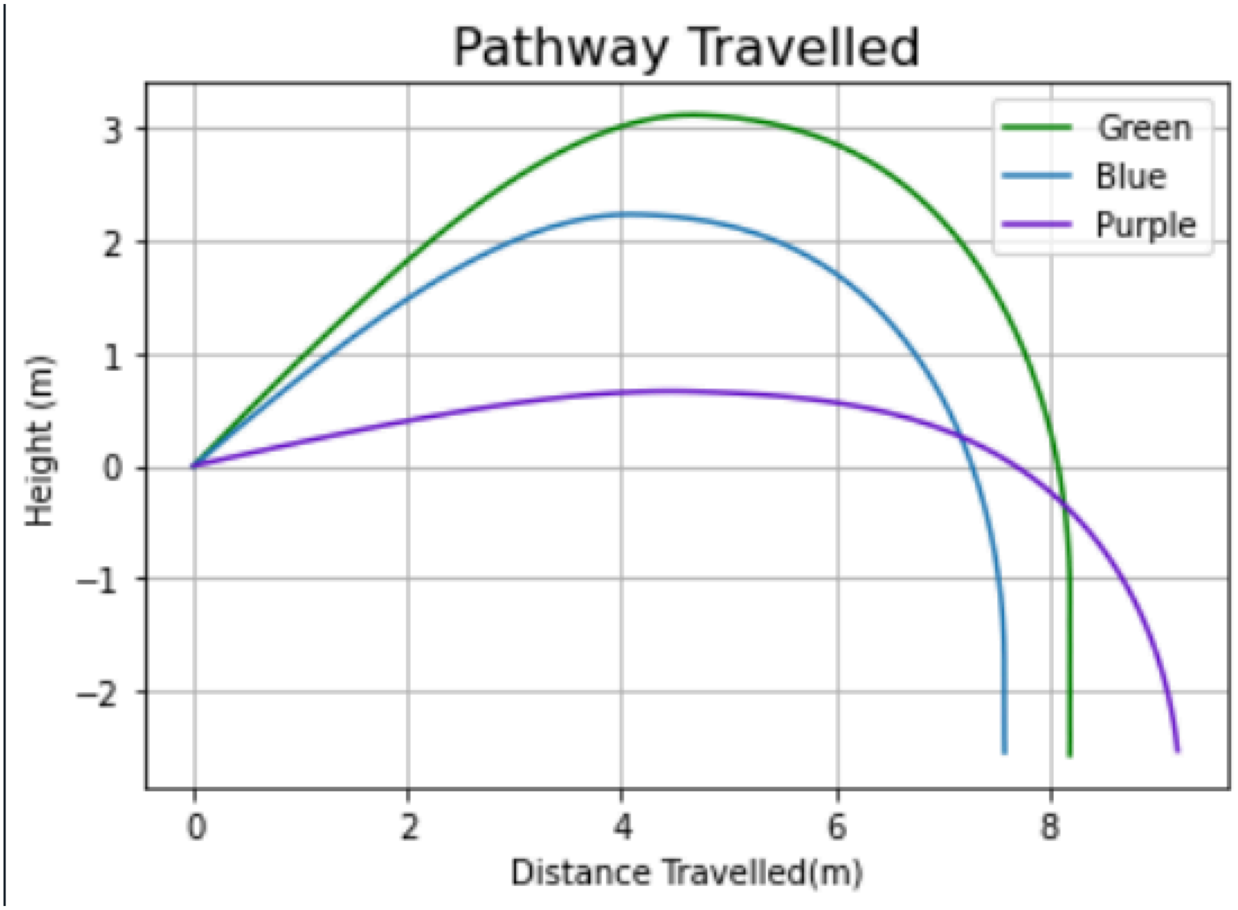


Fig 18: My corresponding calculated trajectories to compare with three of Cohen's observed trajectories reproduced in Fig. 17.

Using my python code, I have graphed three of Cohen's equations (fig.17). The green corresponds to green (v=24.71 m/s, θ= 44°), blue to blue (v= 19.8 m/s, θ= 39°), and magenta to purple (v= 32.3 m/s, θ= 12°). My graph (fig. 18) shows that the trajectory highly varies depending on the launch angle and speed. As one may observe, if Cohen's graph is taken to be the accurate result, a notable distinction becomes evident. Cohen's graph exhibits trajectories that are shorter in distance than those depicted by my Python-generated graph. This discrepancy could be attributed to several factors inherent to my calculations.

The method employed to calculate and update the drag force in the simulation might contribute to the disparities. The drag force, being a function of velocity squared and influenced by factors such as air density and surface area, can be sensitive to variations. A minor difference in the drag force calculations between the two approaches could lead to the observed variation in trajectory lengths. Considering the starting velocity and angle was the same, we might consider that the integration technique was different, as the Euler method includes some level of approximation. The cumulative effect of approximation over multiple time steps can contribute to a longer trajectory in the Python-generated graph. Also, Cohen does not specify how their trajectories were generated, just that they are numerical solutions of the equation of motion. Cohen could have also considered factors such as buoyancy.

In order to match my calculated distances with Cohen's, the drag coefficient was tuned within the range of the literature values I found. The results are reported in Table-D.1 and D.2. In this Table, it must be noted that the launch distance is taken as the distance the ball traveled from the net in the middle for ease of measurement. The raw experimental distance requires all values to increase by 6.71 m. This need for a tuned drag may be for many reasons: Firstly, there may be a different drag coefficient than anticipated by other research. A higher drag coefficient leads to stronger air resistance. This can cause the badminton shuttlecock to slow down more quickly, resulting in a shorter flight distance and altered trajectory compared to a simulation with a lower drag coefficient. In addition, many of the values are quite low. At lower speeds, the flow around the shuttlecock is more likely to be laminar (smooth and ordered). This can result in less turbulent wake and potentially lower drag than expected.

Table D.1: Tuned drag coefficient for length of launch with starting speed created by 60 Psi

Launch Angle	Launch Distance (m) ± 0.01			Average	Stand. Dev	Calculated Distance in Python	Tuned drag coefficient.
	R1	R2	R3				
10.0	3.32	3.02	3.35	3.23	0.4	2.52	0.4
20.0	4.52	4.43	4.34	4.43	0.1	3.63	0.4
30.0	4.03	4.55	4.14	4.24	0.3	3.41	0.3
40.0	4.94	4.11	4.90	4.65	0.5	4.02	0.5
50.0	5.91	4.62	5.16	5.23	0.7	4.23	0.4

60.0	5.56	4.01	3.75	4.44	1.0	4.01	0.5
70.0	3.69	5.12	5.38	4.73	1.0	4.36	0.4
80.0	5.38	5.44	3.94	4.92	0.7	4.57	0.3

Table D.2: Tuned drag coefficient for length of launch with starting speed 44m/s.

Angle	Launch Speed(m/s)	Average Horizontal displacement from net (m). Total displacement requires adding 6.71 m ± 0.01	Stan. Dev.	Calculated Distance In Python	Tuned Cd
45	44	2.65	0.8	16.12	0.5
30	44	3.55	1.1	16.94	0.5
20	44	3.73	0.3	17.14	0.5
15	44	3.98	0.4	17.43	0.5

VI.2- Comparison with experimental Data and Experimental Error

The experimental data is loaded into Python (Fig-19) and values extracted into lists x and y enabling it to be plotted and compared with the calculated trajectory.

```
csv_path = "Send_to_Hallvard1.csv"

rows = []
with open(csv_path) as f:
    reader = csv.reader(f)
    rows = [row for row in reader]


x = []
y = []
spl_char = ";"
num = 0
print("here")
for row in np.array(rows).T:
    for row2 in row:
        a = row2.find(spl_char)
        val = float(row2[0:a])
        x.append(val)
print(x)


for row in np.array(rows).T:
    for row2 in row:
        a = row2.find(spl_char)
        val = row2[a:]
        final_val = float(val.replace(";", ""))
        y.append(final_val*vertical_scale_factor)
print(y)

plt.plot(x, y,label= "Experimental Model")
plt.legend(loc="upper right")
```

Figure 19: The code for CSV File reading

It is therefore possible to look at the experimental and calculated trajectories next to each other to observe the accuracy (fig-20).

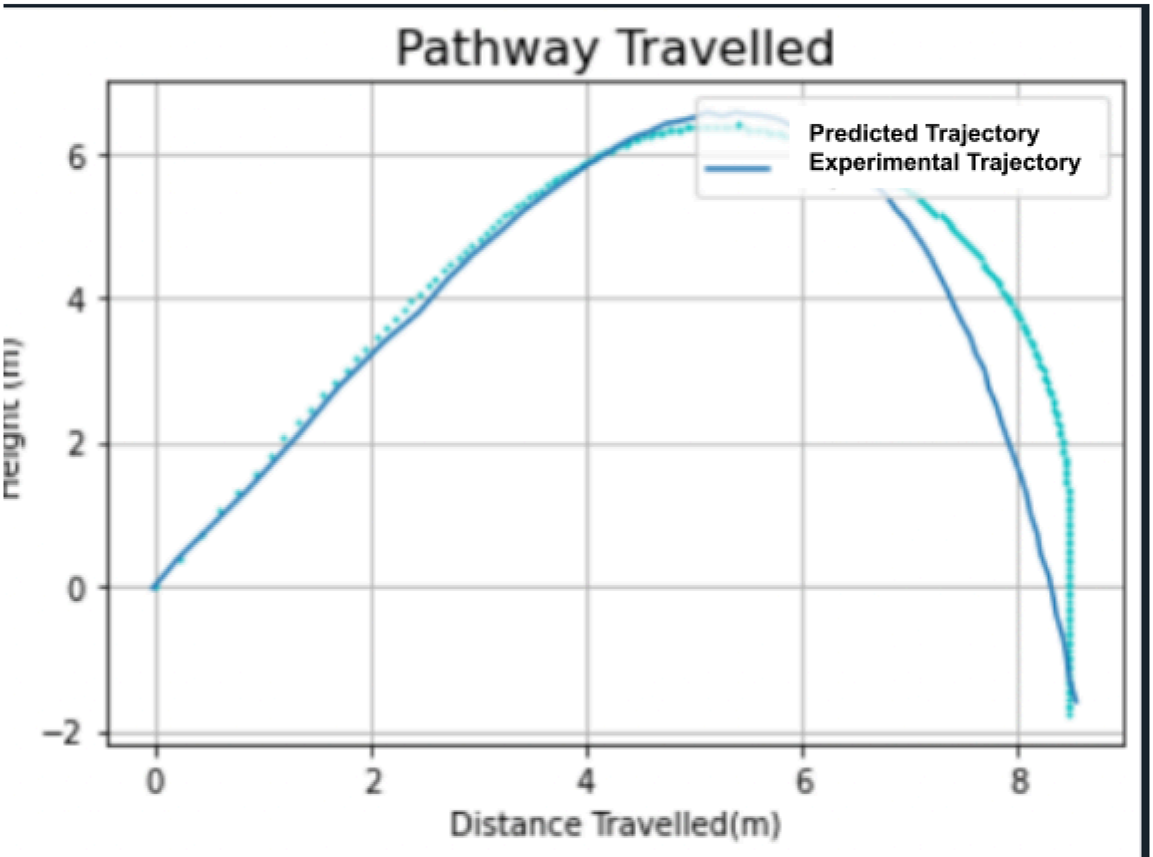


Figure 20: The Predicted Trajectory compared to the Experimental Trajectory in Python

The graph above also had a scale correction factor. The correction factor multiplies all the values by 1.45 to anti-count the filming project angle.

The results also show that the aerodynamic wall explained by Cooke & Firoz is not present in experimental data. In the graph, it is observed that the light blue line has a sudden steep drop, whereas the darker blue line drops more quadratically. This is explained by Reynold's number and that the drag exponent may vary between 1 and 2 as well. This is a limitation to the code, as I did not take the relationship between Rynold number and drag coefficient into account.

There are also other limitations in this experiment. The calculated trajectories do not include many factors to match the measured values. For example, the flip that the projectiles usually do is missing, which may impact the trajectory of a badminton ball. Due to the loss in kinetic energy, this may have made the theoretical range shorter. The data is somewhat lacking due to being done on two types of less than optimal equipment, and the impacts are unknown. There are also sources of error in the assumptions that were made, such that the velocity is constant for the first few seconds (as the instantaneous velocity was calculated by plotting two points close by), which means the measured value for velocity will be smaller than the real initial value.

VII - CONCLUSION

Despite the uncertainty for relatively large or small initial velocity, the model implementation (MI) of Newton mechanics is able to replicate a feasible trajectory depending on the launch angle speed. Data analysis from the experiment shows that for the clear, an angle between 40-50 gave the longest range and was also effective when considering air resistance. This is interesting to consider as it allows further calculations on the most optimal hitting angle depending on the range needed. It also shows that the higher the launch speed, the more clear the phenomena called the aerodynamic wall occurs. The MI and the experimental results fit somewhat well, which means the MI is reliable, but the statements are only acceptable as a rough approximation. The applicability of the MI is to many other types of motion as well, such as smashes or dropshots. However, the MI neglects to consider flips/rotations, which means that it cannot model netspins or other trajectories that include flips. This brings questions for further research.

The investigation finds that depending on the drag force exponent, the trajectory can be described either analytically and/or numerically. When the drag exponent is 1, an analytical solution shown by Eq.6 and Eq. 7 is also available. The equations illustrate that there will always be a decay in the horizontal direction, and that it will eventually reach a minimum speed of zero, before increasing in the other direction until terminal velocity. This allows us to predict a trajectory. When the drag exponent is 2, the trajectory can only be calculated numerically with a numerical method and Python code. This answers the research question, as it exhibits the similar trajectory an shuttle would be expected to have.

The dynamic system which this essay has analyzed is crucial in explaining the mechanics of other similar systems such as the motion through normal air of any other projectile. The MI may also be applicable to finding the optimal hitting angle for a high clear in a badminton game on the basis of being higher than a certain interception height with the shortest travel time.

VIII - SOURCES

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IX- APPENDIX

Appendix A:

$$m\frac{dv_x}{dt} = - bv_x$$

$$\frac{dv_x}{v_x} = - b\frac{dt}{m}$$

$$\int \frac{dv_x}{v_x} = \int \frac{-b \times dt}{m}$$

$$\therefore \ ln\, v_x = \frac{-b}{m}t + c$$

$$v_x = e^c \times e^{\frac{-b}{m}t}$$

$$v_x = c \times e^{\frac{-b}{m}t}$$

$$\frac{dx}{dt} = c \times e^{\frac{-b}{m}t}$$

$$\int dx = \int c \times e^{\frac{-b}{m}t} \times dt$$

$x = - \frac{c\times}{m}e^{\frac{-b}{m}t} + c_1$ (c_1 indicates a different constant)

$\therefore$

$$X(t) = c_1 + \frac{cm}{b} \times e^{\frac{-b}{m}t}$$

However, previously we defined $v_x = c \times e^{\frac{-b}{m}t}$

So it is actually $X(t) = c_1 + v_0\frac{m}{b} \times e^{\frac{-tb}{m}}$

Also, if we use the starting displacement t=0 and x=0,

$0 = c_1 + \frac{v_0 m}{b}$

$\therefore c_1 = - \frac{v_0 m}{b}$

However, we might not start from that displacement so $X(t) = x_0 + v_0 \frac{m}{b}(1 - e^{\frac{-tb}{m}})$

And solving for y gives:

$$m\frac{dv_y}{dt} =- mg - bv_y$$

$$u = V_y + \frac{mg}{b}$$

So: $- bu =- mg - bV_y$

$$m\frac{du}{dt} =- bu$$

So we integrate to get: $ln\, u = \frac{-b}{m}t + c$ and $u = V_y + \frac{mg}{b}$

$$\therefore u = e^{\frac{-b}{m}t + c}$$

$$V_y + \frac{mg}{b} = c_3 e^{\frac{-b}{m}t}$$

$$V_y = c_3 e^{\frac{-b}{m}t} - \frac{mg}{b}$$

$$\frac{dy}{dt} = c_3 e^{\frac{-b}{m}t} - \frac{mg}{b}$$

$$\int dy = \int (c_3 e^{\frac{-b}{m}t} - \frac{mg}{b}) \times dt$$

$$y =- \frac{c_3 m e^{\frac{-b}{m}t}}{b} - \frac{mgt}{b} + c_4$$

Now let c_3 and c_4 be defined using: ($t=0$, $V_y = V_{y0}$) and ($t=0$, $y=0$).

So when $V_y = c_3 e^{\frac{-b}{m}t} - \frac{mg}{b}$, $t=0$, $V_y = V_{y0}$. This means that $V_{y0} = c_3 \times 1 - \frac{mg}{b}$

$$\therefore c_3 = \frac{mg}{b} + V_{y0}$$

And using $y =- \frac{c_3 m e^{\frac{-b}{m}t}}{b} - \frac{mgt}{b} + c_4$,

It derives: $0 = \frac{c_3 \times m}{b} + c_4$

$$\therefore c_4 =- \frac{mc_3}{b}$$

Hence the equation $y = -\frac{c_3 m e^{\frac{-b}{m}t}}{b} - \frac{mgt}{b} + c_4$ becomes:

$$y = -\frac{c_3 m e^{\frac{-b}{m}t}}{b} - \frac{mgt}{b} - \frac{mc_3}{b}$$

$$y = -\frac{mc_3}{b}(e^{\frac{-b}{m}t} - 1) - \frac{mgt}{b}$$

$$\frac{mg}{b} = \ terminal\ velocity\ (V_T)$$

The equation therefore equals =

$y = -\frac{m(\frac{mg}{b} + V_{y0})}{b}(e^{\frac{-b}{m}t} - 1) - \frac{mgt}{b}$ (where the last term is the starting velocity)

$$y = y_0 + V_T + (e^{\frac{-tb}{m}} - 1)(V_T \frac{m}{b} - \frac{V_0 m}{b})$$